Gravity: Overview

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"Fundamental Physics with LISA" workshop,
GGI, 12 November 2018

Beyond Einstein Theories of Gravity

Type I: **UV Modifications:**

eg. Quantum Gravity, string theory, extra dimensions, branes, supergravity

At energies well below the scale of new physics Λ , gravitational effects are well incorporated in the language of Effective Field Theories

$$S = M_{\rm Planck}^2 \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{a}{\Lambda^2}R^2 + \frac{b}{\Lambda^2}R_{\mu\nu}^2 + \dots + \frac{c}{\Lambda^4}R_{abcd}R^{cd}_{ef}R^{efab} + \dots + \mathcal{L}_{\rm matter}\right]$$
$$+\frac{d}{\Lambda^6}(R_{abcd}R^{abcd})^2 + \dots$$

eg Cardoso et al 2018

Addition of Higher Dimension, (generally higher derivative operators), **no** failure of well-posedness/ghosts etc as all such operators should be treated perturbatively (rules of EFT)

Type 2: **IR Modifications:**

Why modify gravity (in the IR)?

Principle Motivation is Cosmological:

Dark Energy and Cosmological Constant

I: Old cosmological constant problem:

Why is the universe not accelerating at a gigantic rate determined by the vacuum energy?

II: New cosmological constant problem:

Assuming I is solved, what gives rise to the remaining vacuum energy or dark energy which leads to the acceleration we observe?

Why modify gravity (in the IR)?

III: Because it allows us to put better constraints on Einstein gravity!

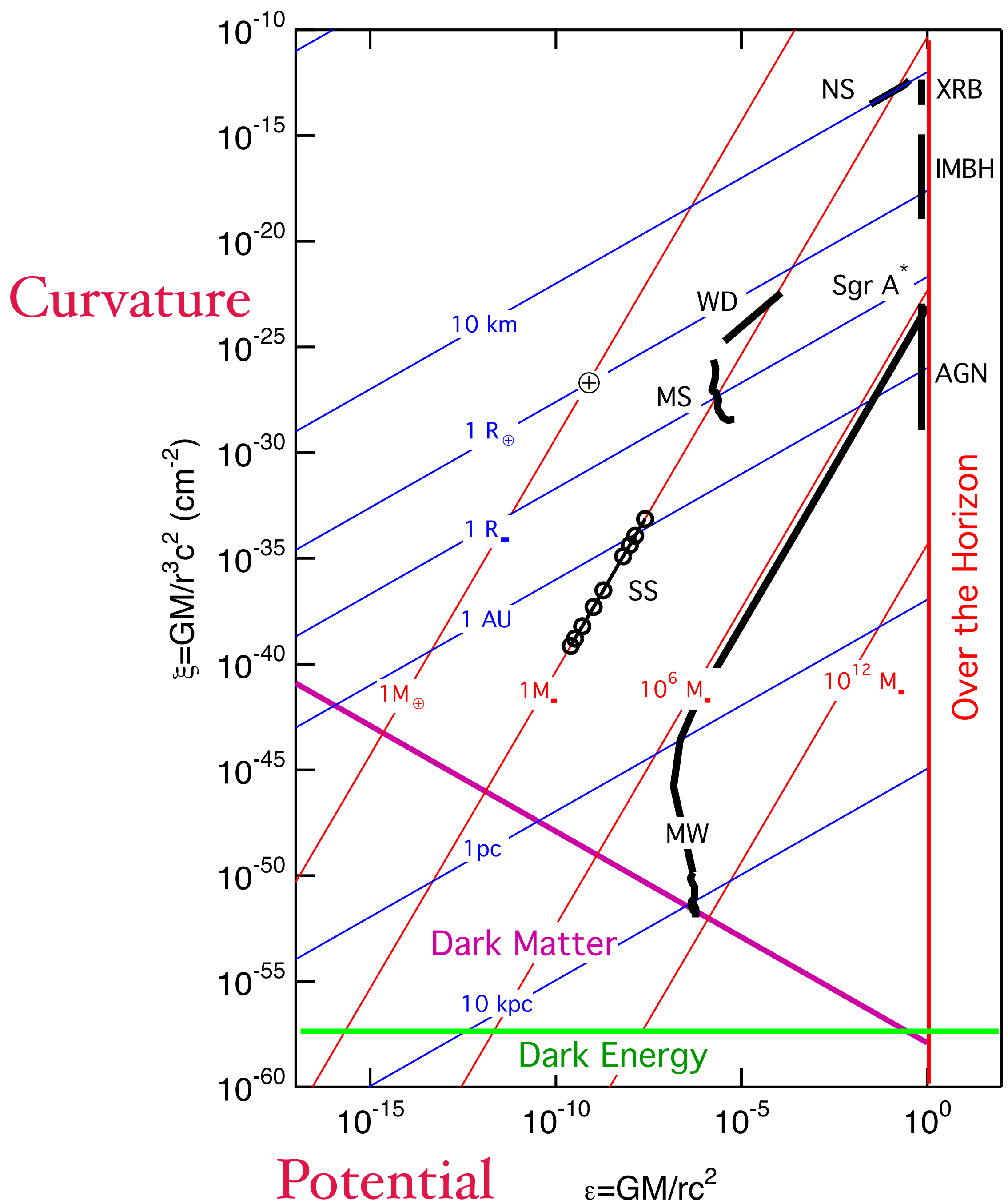


Figure 1: A parameter space for quantifying the strength of a gravitational field. The x-axis measures the potential $\epsilon \equiv GM/rc^2$ and the y-axis measures the spacetime curvature $\xi \equiv GM/r^3c^2$ of the gravitational field at a radius r away from a central object of mass M. These two parameters provide two different quantitative measures of the strength of the gravitational fields. The various curves, points, and legends are described in the text.

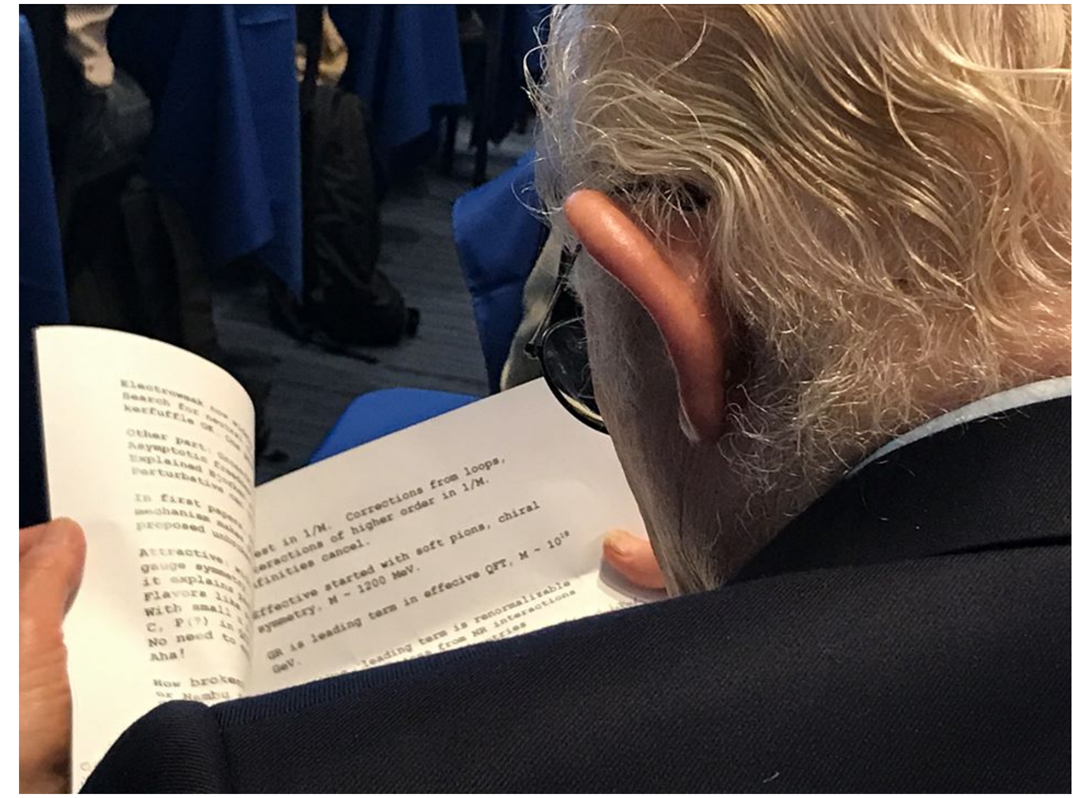
Gravity has only been tested over special ranges of scales and curvatures

e.g. Weinberg's nonlinear Quantum Mechanics- constructing to test linearity of QM

D. Psaltis, Living Reviews

Guiding Principle

Theorem: General Relativity is the Unique local and Lorentz invariant theory describing an interacting single massless spin two particle that couples to matter



Weinberg, Deser, Wald, Feynman, …

Locality

Massless ? Lorentz Invariant

Single Spin 2

Guiding Principle

Theorem: General Relativity (with a c.c.) is the Unique local and Lorentz invariant theory describing an interacting single massless spin two particle that couples to matter

Weinberg, Deser, Wald, Feynman, ...

Locality ✓

Massless ✓

?

Lorentz Invariant ✗

Thomas - Lorentz violation

Single Spin 2 ✗

Filippo - Additional Scalar

Guiding Principle

Corollary: Any theory which preserves Lorentz invariance and Locality leads to new degrees of freedom!

Locality ✓

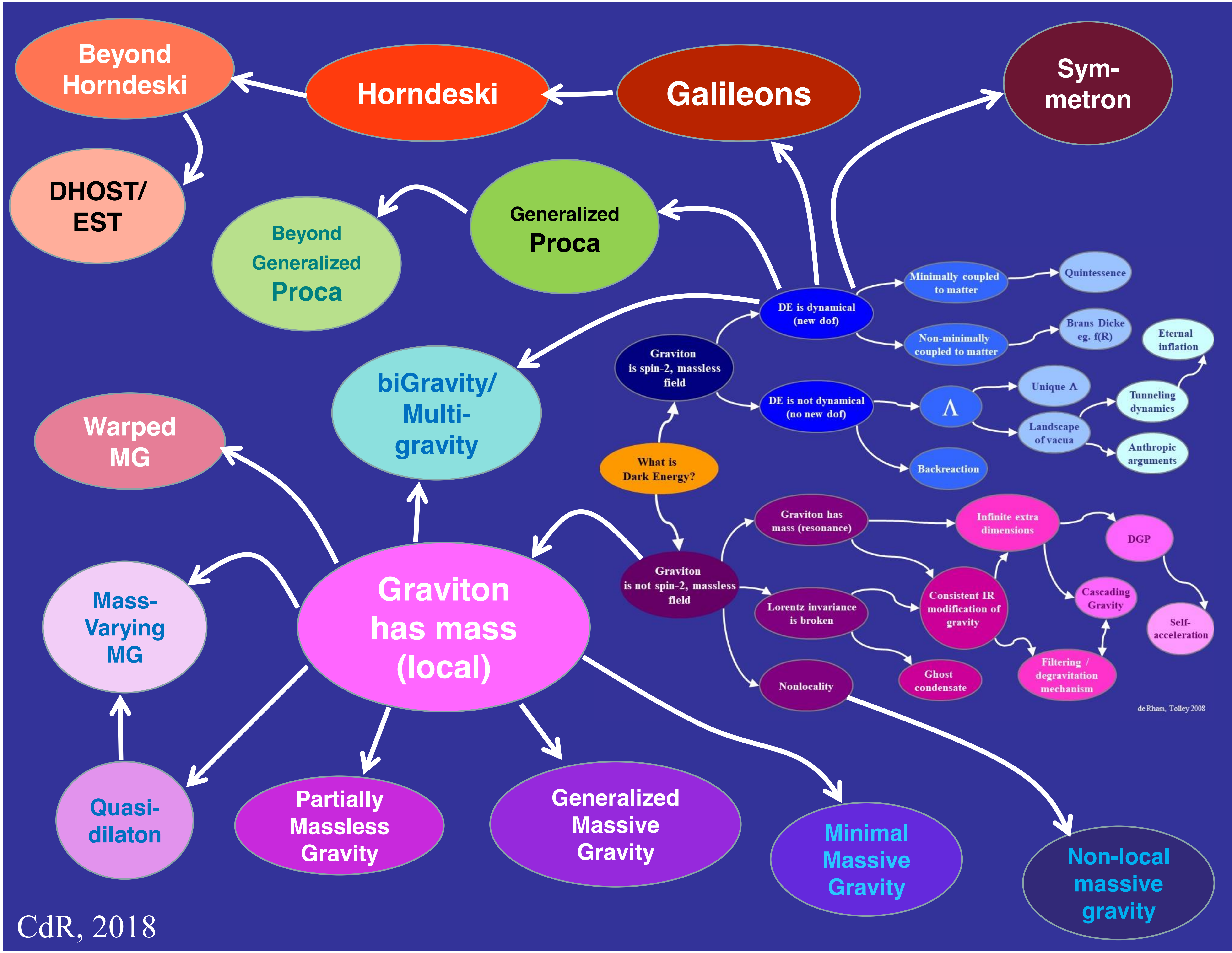
Massless ✗

?

Lorentz Invariant ✓

Andrew - massless and/or massive spin 2 states

Single Spin 2 ✗



Beyond Horndeski
Horndeski
Galileons
Sym-metron
DHOST/ EST
Beyond Generalized Proca
Generalized Proca
Minimally coupled to matter
Quintessence
DE is dynamical (new dof)
Non-minimally coupled to matter
Brans Dicke eg. f(R)
Eternal inflation
Graviton is spin-2, massless field
biGravity/ Multi-gravity
Unique Λ
Λ
Tunneling dynamics
DE is not dynamical (no new dof)
Landscape of vacua
Warped MG
Anthropic arguments
What is Dark Energy?
Backreaction
Graviton has mass (resonance)
Infinite extra dimensions
DGP
Graviton is not spin-2, massless field
Consistent IR modification of gravity
Cascading Gravity
Mass-Varying MG
Graviton has mass (local)
Lorentz invariance is broken
Self-acceleration
Ghost condensate
Filtering / degravitation mechanism
Nonlocality
de Rham, Tolley 2008
Quasi-dilaton
Partially Massless Gravity
Generalized Massive Gravity
Minimal Massive Gravity
Non-local massive gravity
CdR, 2018

New gravitational degrees of freedom that coupled to matter are highly constrained: Need - Screening Mechanisms

Imagine a scalar $\phi = \phi_b + \delta\phi$

coupled to the energy density $\rho = \rho_b + \delta\rho$

Generic form of equation of motion for perturbations

$$Z(\phi_b, \rho_b)\left[\frac{d^2\delta\phi}{dt^2} - c_s^2\frac{d^2\delta\phi}{dx^2}\right] + m^2(\phi_b, \rho_b)\delta\phi = \beta(\phi_b, \rho_b)G_{\mathrm{Newton}}\delta\rho$$

kinetic term

mass term

coupling to matter

gradient term

Fifth force constraints: screening

$$F \approx \frac{M_a M_b G}{r^2} \frac{\beta^2(\phi_b, \rho_b)}{\sqrt{Z}(\phi_b, \rho_b) c_s(\phi_b, \rho_b)} \exp(-m(\phi_b, \rho_b) r)$$

To ensure fifth forces are small

$$\frac{\beta^2(\phi_b, \rho_b)}{\sqrt{Z(\phi_b, \rho_b)} c_s(\phi_b, \rho_b)} \exp(-m(\phi_b, \rho_b) r)$$

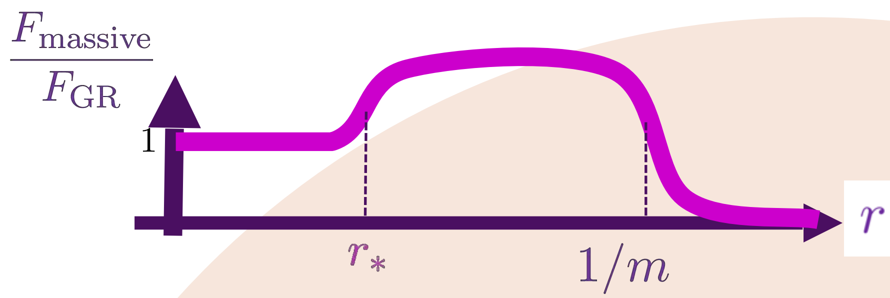
Only three independent possibilities!

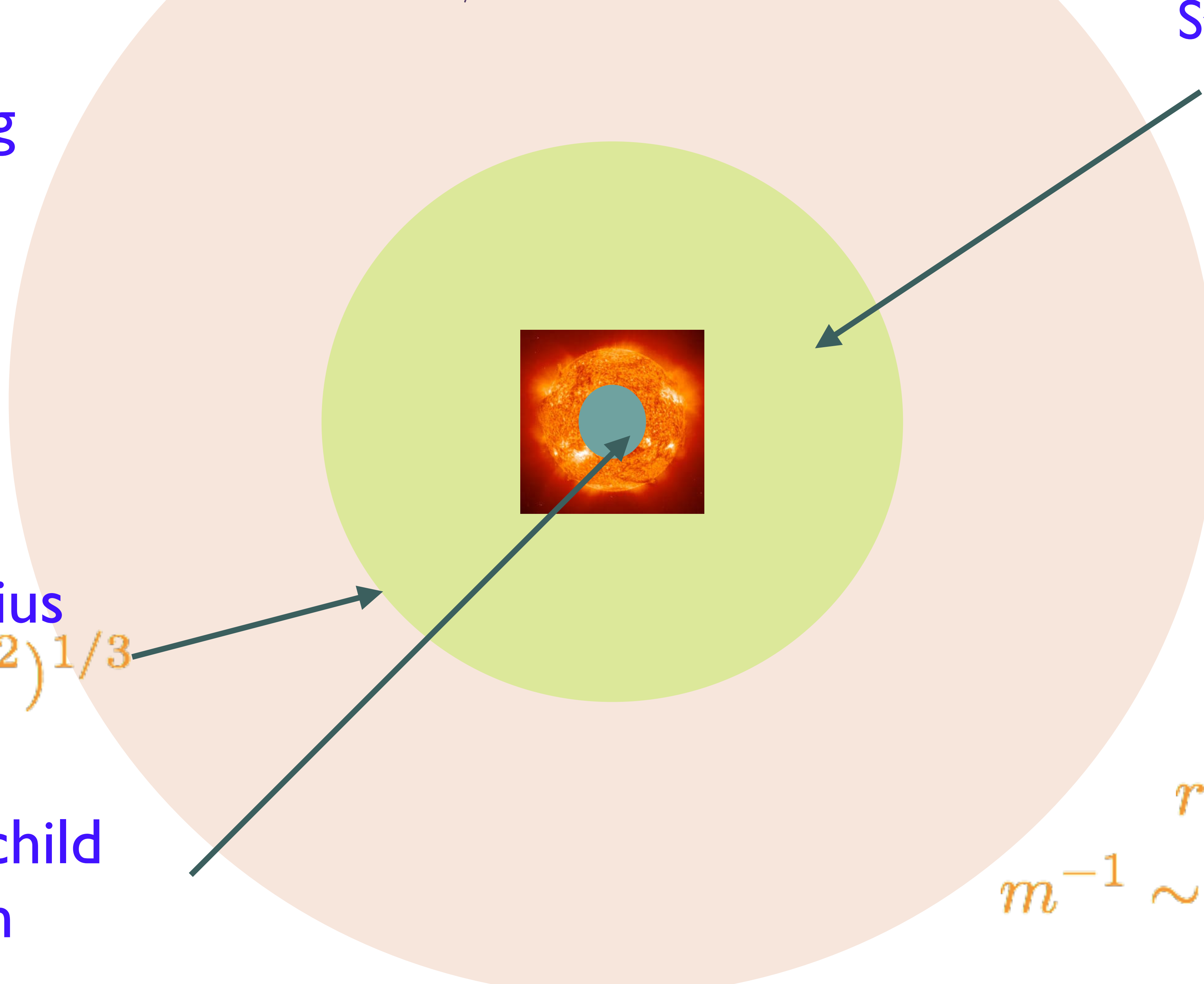
(a) Coupling is small $\beta(\phi_b, \rho_b) \ll 1$ Symmetron

(b) Mass is large $m(\phi_n, \rho_b) \gg \frac{1}{r_{exp}}$ Chameleon

(c) Kinetic term is large $Z(\phi_b, \rho_b) \gg 1$ Vainshtein

Vainshtein effect is strongly scale and density dependent





Yukawa region

$r > m^{-1}$

Strong coupling region

$r \ll r_V$

$Z \gg 1$

Weak coupling region

$r \gg r_V$

$Z \sim 1$

Vainshtein radius

$r_V = (r_s m^{-2})^{1/3}$

Schwarzschild region

$r < r_s$

For Sun

$r_V \sim 250pc$

$m^{-1} \sim 4000Mpc$

$r_s \sim 3km$

Massive Gravity: Hard or Soft?

Hard

A generic local, Lorentz invariant theory at the linearized level gives the following interaction between two stress energies

$$A \sim \frac{1}{M_{\rm Pl}^2}\int \frac{d^4k}{(2\pi)^4} T^{ab}(k)^* \left[\frac{P_{abcd}}{k^2} + \sum_{\rm pole} Z^{(2)}_{\rm pole} \frac{\mathcal{P}_{abcd}}{k^2+m^2_{\rm pole}} + \sum_{\rm pole} Z^{(0)}_{\rm pole} \frac{\eta_{ab}\eta_{cd}}{k^2+m^2_{\rm pole}} \right] T^{cd}(k)$$

$$+\frac{1}{M_{\rm Pl}^2}\int \frac{d^4k}{(2\pi)^4} T^{ab}(k)^* \left[\int d\mu\, \rho^{(2)}(\mu) \frac{\mathcal{P}_{abcd}}{k^2+\mu^2} + \rho^{(0)}(\mu)\frac{\eta_{ab}\eta_{cd}}{k^2+\mu^2}\right] T^{cd}(k)$$

Soft

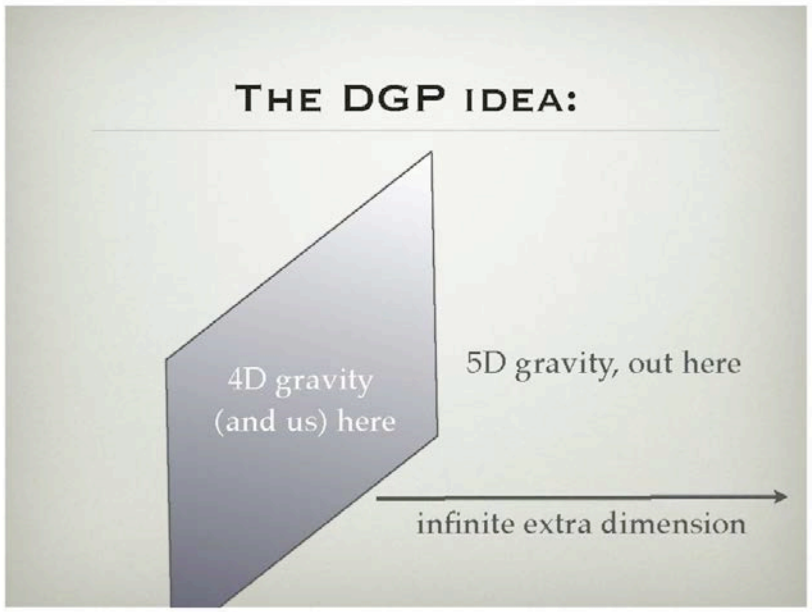
$$P_{abcd} = \eta_{ac}\eta_{bd} + \eta_{bc}\eta_{ad} - \eta_{ab}\eta_{cd}$$

$$\mathcal{P}_{abcd} = \eta_{ac}\eta_{bd} + \eta_{bc}\eta_{ad} - \frac{2}{3}\eta_{ab}\eta_{cd}$$

Soft Massive Graviton is a **resonance**

Hard Massive Graviton is a **pole** (infinite lifetime)

Soft Massive Gravity: DGP Model



Soft Massive Gravity theories were constructed first!
Naturally arise in Braneworld Models: **DGP**,
Cascading Gravity: Soft Massive Graviton is a
<u>Resonance State</u> localized on Brane

$$\Delta S \sim \frac{1}{M_{\text{Planck}}^2} \int \frac{d^4k}{(2\pi)^4} T^{\mu\nu}(k) \left[\int_0^\infty d\mu \frac{\rho(\mu) P_{\mu\nu\alpha\beta}(k)}{k^2 + \mu} \right] T^{\alpha\beta}(k)$$

Soft

More irrelevant

More relevant

$$S = \int d^4x \sqrt{-g_4} \frac{M_4^2}{2} R_4 + \int d^4x \sqrt{-g_4} \mathcal{L}_M + \int d^5x \sqrt{-g_5} \frac{M_5^3}{2} R_5$$

Dominates in UV

Dominates in IR

Hard Λ_3 Massive Gravity

$$Diff(M) \times \text{Poincare} \to \text{Poincare}_{\text{diagonal}}$$

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2\, R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n \right) + \mathcal{L}_M$$

$$K = 1 - \sqrt{g^{-1} f}$$

$$\text{Det}[1+\lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Characteristic Polynomials

Unique low energy EFT where the strong coupling scale is

$$\Lambda_3 = (m^2 M_P)^{1/3}$$

5 propagating degrees of freedom

5 polarizations of gravitational waves!!!!

Hard Massless plus Λ_3 Massive Gravity

$$\mathcal{L} = \frac{1}{2}\left(M_P^2\sqrt{-g}R[g] + M_f^2\sqrt{-f}R[f] - m^2\sum_{n=0}^{d}\beta_n U_n(K)\right) + \mathcal{L}_M$$

$$\mathrm{Det}[1+\lambda K] = \sum_{n=0}^{d}\lambda^n \mathcal{U}_n(K)$$

$$K = 1 - \sqrt{g^{-1}f}$$

decoupling limit

$$\Big\downarrow \quad M_f \to \infty$$

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2\sum_{n=0}^{4}\beta_n \mathcal{U}_n\right) + \mathcal{L}_M$$

Bigravity=
massless graviton (2 d.o.f.)
+ massive graviton (5 d.o.f.)

+decoupled massless graviton $f_{\mu\nu}$

Universal Decoupling Limit: Galileon

At energies $m \ll E \ll M_{\text{PLanck}}$ $\Lambda_3 = (m^2 M_{\text{Planck}})^{1/3}$

All Lorentz invariant Hard and Soft and Multi-graviton theories look like **Galileon theories** (plus massless spin 2 plus Maxwell)

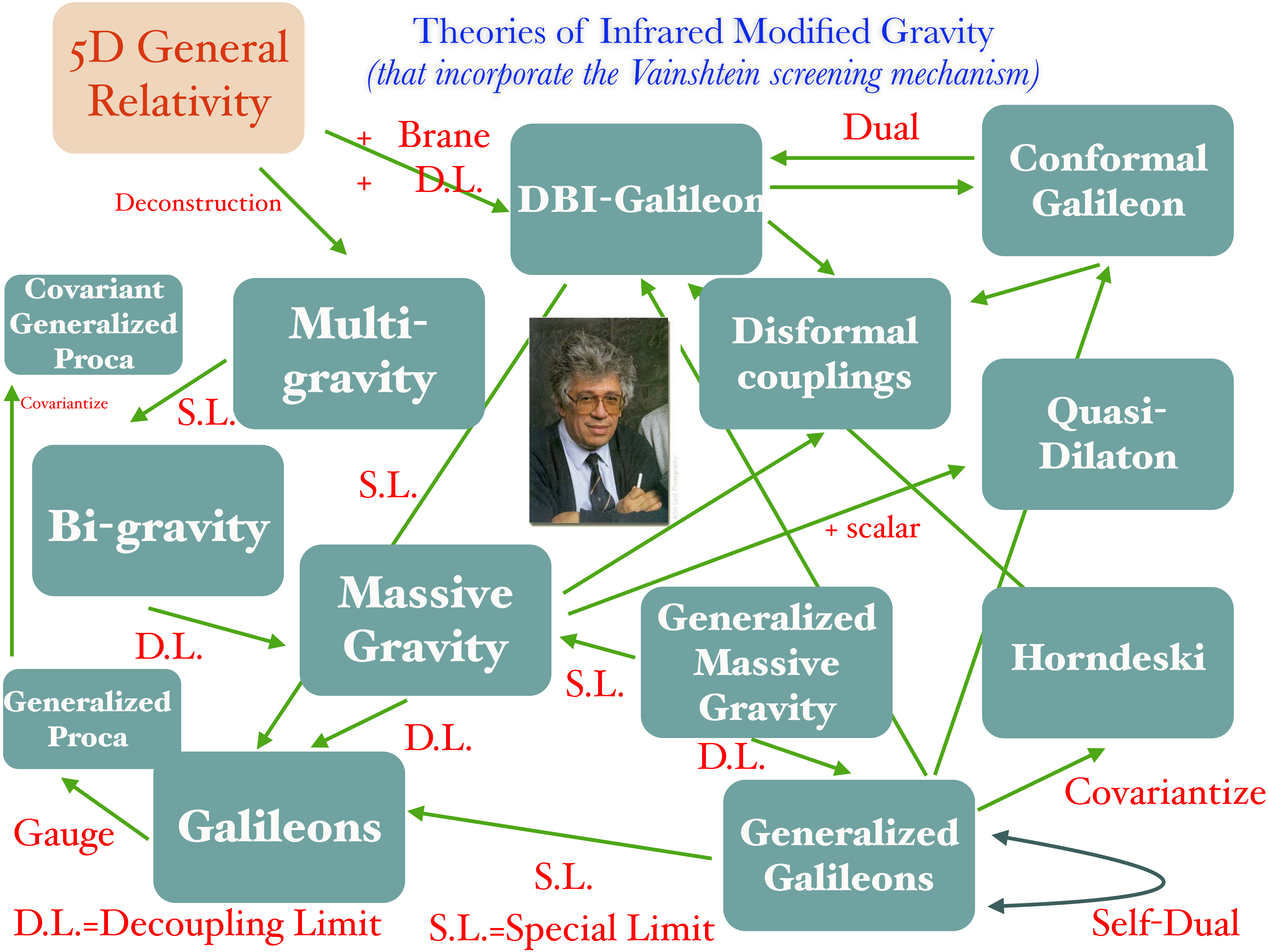
$$\pi \to \pi + v_\mu x^\mu + c \qquad K_{\mu\nu} = \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$$

$$S = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} v^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} v_{\alpha\beta} \right] + S_{\text{Galileon}} + S_{\text{mattercoupling}}$$

$$S_{\text{Galileon}} = \sum_{n=0}^{4} \pi \, c_n \mathcal{U}_n(K) \qquad \text{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Novel feature, matter has `disformal' couplings

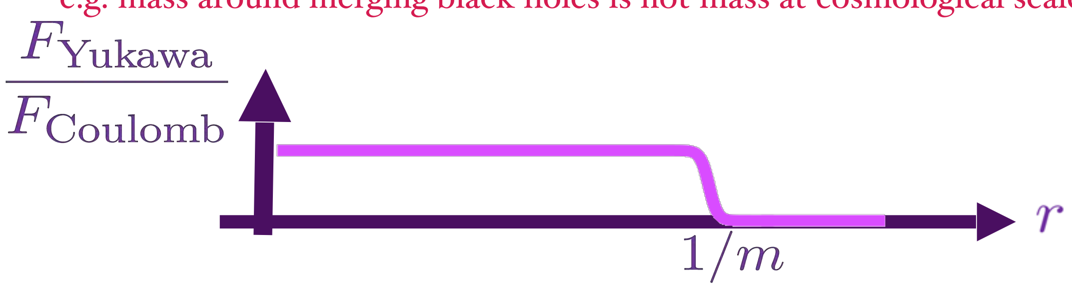
$$S_{\text{matter coupling}} = \int d^4x \frac{1}{M_P} (\pi T + \partial_\mu \pi \partial_\nu \pi T^{\mu\nu} + \dots)$$


Theories of Infrared Modified Gravity
(that incorporate the Vainshtein screening mechanism)
5D General Relativity
+ Brane
+ D.L.
Deconstruction
DBI-Galileon
Dual
Conformal Galileon
Covariant Generalized Proca
Multi-gravity
Disformal couplings
Covariantize
S.L.
Quasi-Dilaton
S.L.
Bi-gravity
+ scalar
Massive Gravity
D.L.
Generalized Massive Gravity
S.L.
Horndeski
Generalized Proca
D.L.
D.L.
Covariantize
Galileons
Gauge
Generalized Galileons
S.L.
D.L.=Decoupling Limit
S.L.=Special Limit
Self-Dual

Constraints on the Graviton Mass

In a realistic interacting theory, mass for tensor modes of graviton depends on the environment

e.g. mass around merging black holes is not mass at cosmological scales!!!!!!

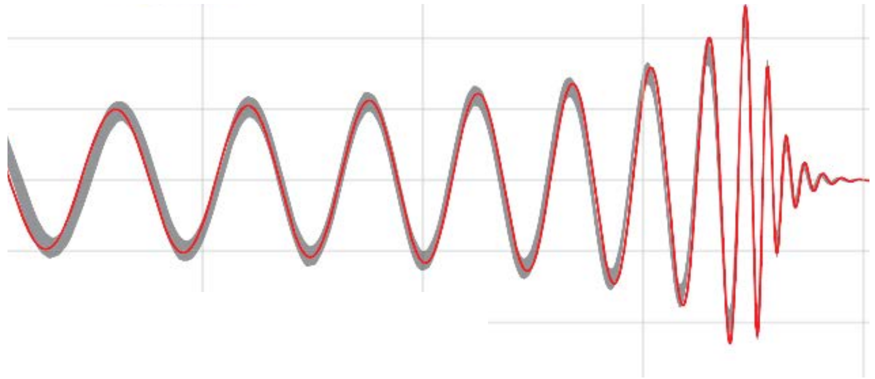


Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters

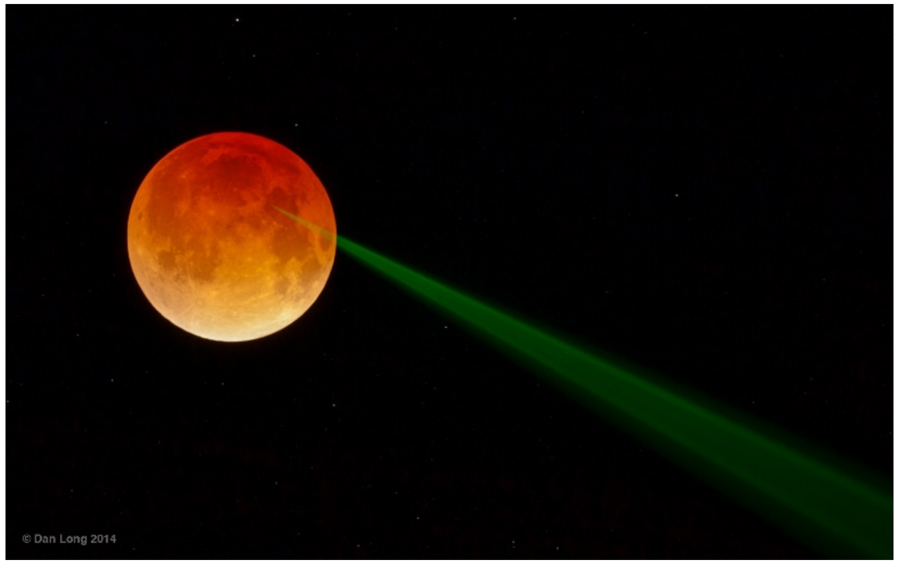
Dispersion Relation

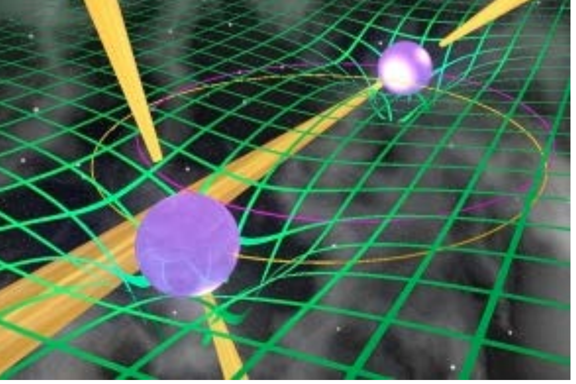
m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB



Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

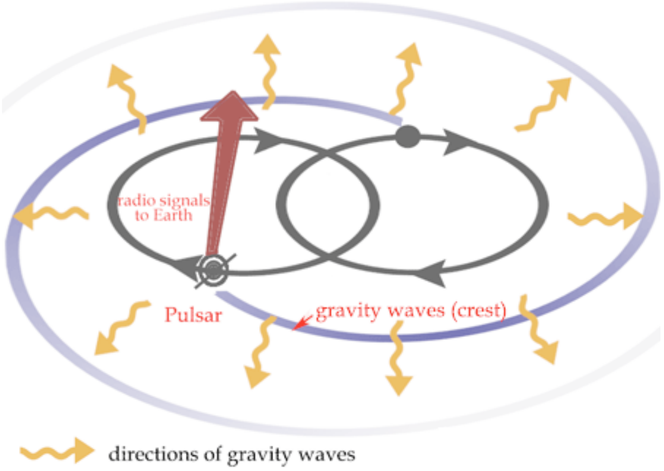




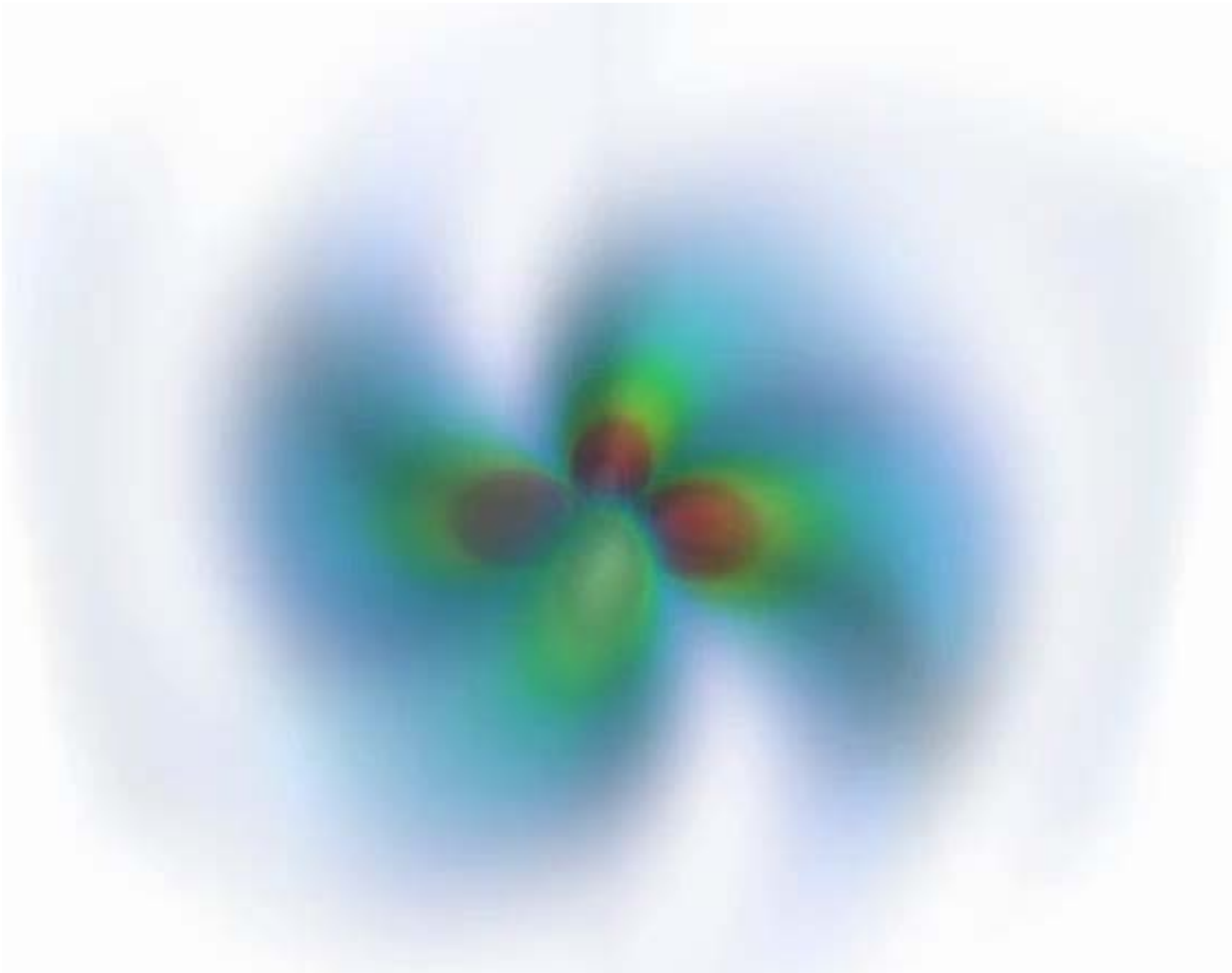
Binary Pulsars

Extra polarizations of graviton = extra modes of gravitational wave

Binary pulsars lose energy faster than in GR so the orbit slows down more rapidly



Dar et al 2018



$$P_2^{\rm cubic} = \frac{M^2}{8\pi M_{\rm Pl}^2}\,\frac{45\times 3^{1/4}\pi^{3/2}}{1024\,\Gamma\left(\frac{9}{4}\right)^2}\,\frac{(\Omega_{\rm p}\bar{r})^3}{(\Omega_{\rm p} r_{\rm v})^{3/2}}\Omega_{\rm p}^2$$

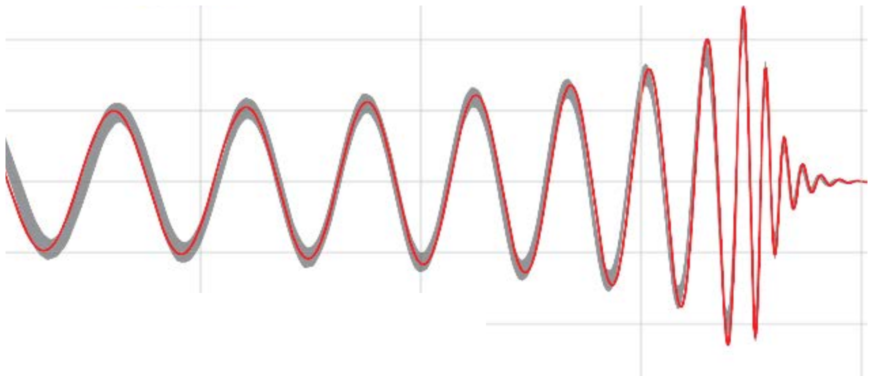
$$\frac{P_2^{\rm cubic}}{P_2^{\rm KG}} = \frac{25\times 3^{17/4}\pi^{3/2}}{1024\,\Gamma\left(\frac{9}{4}\right)^2}(\Omega_{\rm p}\bar{r})^{-1}(\Omega_{\rm p} r_{\rm v})^{-3/2}$$

Quadrupole ‘Galileon’ Radiation

At present only worked out for cubic Galileon, not general DL of MG

Direct Detection of GW

Dispersion Relation		
m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

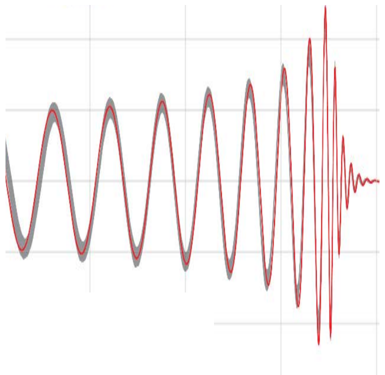


Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity

GW signal would be more squeezed than in GR



Speed increases with frequency

$$v_g/c \approx 1 - \frac{1}{2}(c/\Lambda_g f)^2$$

$$1 - \frac{v_g}{c} = 5 \times 10^{-17} \left(\frac{200\mathrm{Mpc}}{D}\right)\left(\frac{\Delta t}{1\mathrm{s}}\right)$$

$$m_g \lesssim 4 \times 10^{-22}\mathrm{eV} \left(f \Delta t \frac{f}{100\mathrm{Hz}} \frac{200\mathrm{Mpc}}{D}\right)^{1/2}$$

For GW150914,

$D \sim 400\mathrm{Mpc},\ f \sim 100\mathrm{Hz},\ \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\mathrm{eV}$

Will 1998
Abbott et al., 2016

Does we know all the constraints on graviton mass from aLIGO??

$$m_{\text{graviton}} < 10^{-22}\text{eV}$$

No! Many other effects to consider

- Graviton Mass *depends on environment,* for instance it *depends on distance to black holes*
- Graviton Mass likely to vary non-adiabatically during merger creating additional non-adiabatic effects in the waveform
- Additional scalar (and vector) gravitational radiation. Scalar radiation may dominate effects on tensors.
- Black hole/NS solution modified, in particular quasi-normal modes may be different
- Vainshtein suppression may not be active in merger region - needs proper numerical simulation
- PN expansion almost certainly doesn't work in Vainshtein region

Not yet known what the correct asymptotically flat Black Hole Solutions are in Massive Gravity

There should be a solution with Yukawa asymptotics!

= Schwarschild as $m \to 0$

Black Hole Mechanics for Massive Gravitons

Rachel A. Rosen[1]

[1]*Department of Physics, Columbia University, New York, NY 10027, USA*

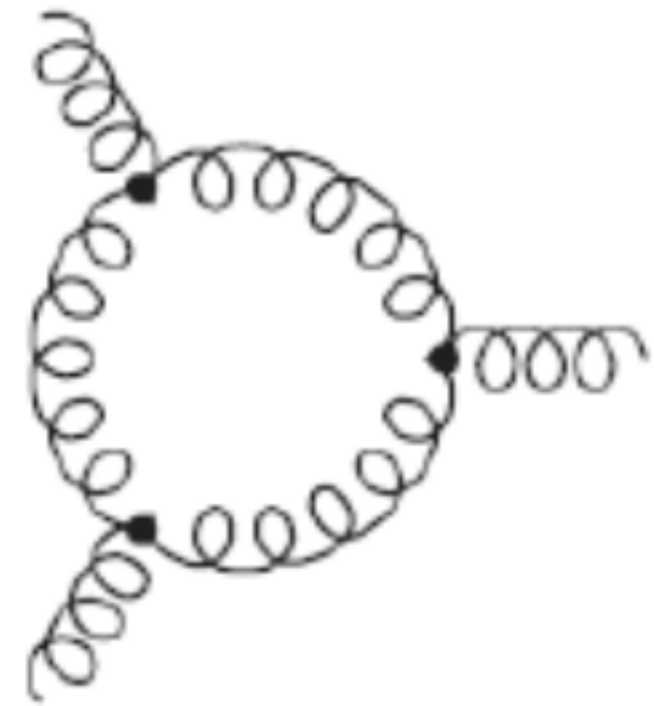
It has been argued that black hole solutions become unavoidably time-dependent when the gr ton has a mass. In this work we show that, if the apparent horizon of the black hole is a null sur with respect to a fiducial Minkowski reference metric, then the location of the horizon is necessa time-*independent*, despite the dynamical metric possessing no time-like Killing vector. This resu non-perturbative and model-independent. We derive a second law of black hole mechanics for t black holes and determine their surface gravity. An additional assumption establishes a zeroth a first law of black hole mechanics. We apply these results to the specific model of dRGT ghost- massive gravity and show that consistent solutions exist which obey the required assumptions. determine the time-dependent scalar curvature at the horizon of these black holes.

Likely real constraints on LI MG are stronger!

Massive Gravity/Galileons etc as an EFT

One-loop Graviton diagram needs counter-terms at the scale

$$K = 1 - \sqrt{g^{-1}f} \qquad \Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$



In decoupling limit: $M_{\rm PLanck} \to \infty, m \to 0$

$$K_{\mu\nu} \to \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$$

EFT corrections then take the form

(even away from the decoupling limit)

de Rham, Melville, Tolley 2017

$$\Lambda^4 L_0 = \left[\frac{M^2}{2} R - \Lambda^3 M \sum_n \alpha_n \mathcal{E}\mathcal{E} g^{4-n} K^n \right] + \Lambda^4 \sum \beta_{p,q,r} \left(\frac{\nabla}{\Lambda} \right)^p K^q_{\mu\nu} \left(\frac{R_{\mu\nu\rho\sigma}}{\Lambda^2} \right)^r$$

<u>Infinite number</u> of derivative suppressed operators

HOW DO WE GO BEYOND THIS?

UV Completion and Causality

Part of a larger question: Are all EFTs allowed?

aka Swampland!

$$[\hat{O}(x), \hat{O}(y)] = 0 \quad \text{if } (x-y)^2 > 0$$

With typical assumption that:

UV completion is Local, Causal, Poincare Invariant and Unitary (Wilsonian)

Answer: NO! Certain EFTs do not admit well defined UV completions

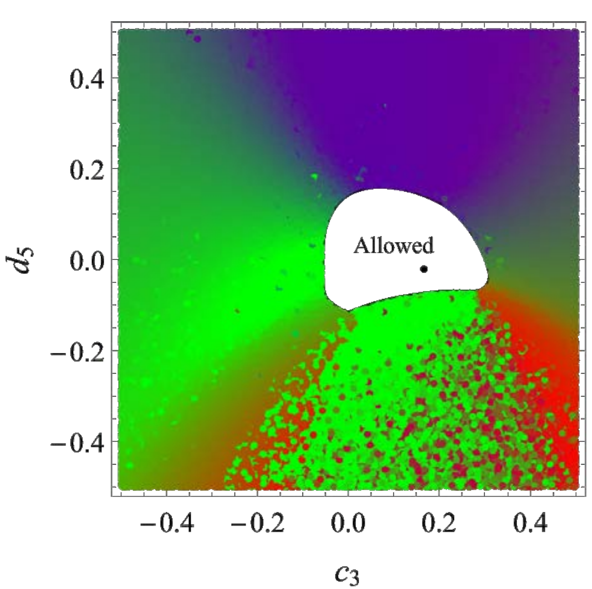
Unitarity + Analyticity: S-matrix Positivity Bounds!

$$\frac{d^2 A(s,t)}{ds^2} > 0$$

$$0 \leq t < 4m^2$$

Causality (Asymptotic (Sub)Luminality): Positive Wigner-Eisenbud time delay

$$T \sim \frac{d\delta(E)}{dE} > 0$$



Constraints parameters of massive gravity/multigravity but so far does not rule it out ...

Guiding Principle

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Weinberg, Deser, Wald, Feynman, ...

Locality

Massless ? Lorentz Invariant

Single Spin 2 $\ddot{a} > 0$

Disclaimer: no references!

Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Quintessence: $$\mathcal{L} = R + V(\phi) - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

$$w = \frac{p_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} \neq -1$$

Scalar-tensor theories

Simplest models of modified gravity are base on single scalar field (universal coupling)

Quintessence: $\mathcal{L} = R + V(\phi) - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

k-essence: $\mathcal{L} = R + G_2(\phi, X)\ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

$c_s^2 \neq 1$: clustering

Scalar-tensor theories

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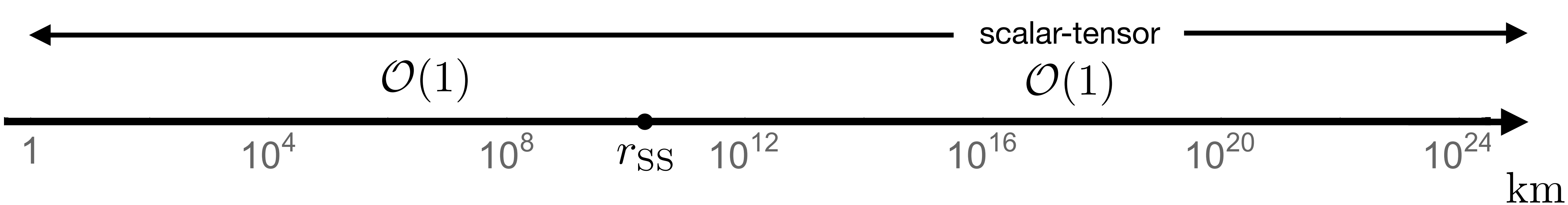
k-essence: $\mathcal{L} = R + G_2(\phi, X)\ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Scalar-tensor: $\mathcal{L} = f(\phi)R + G_2(\phi, X)$

$$G_{\mu\nu}^{(\mathrm{modified})} = 8\pi G\left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\phi)}\right)$$

self-acceleration



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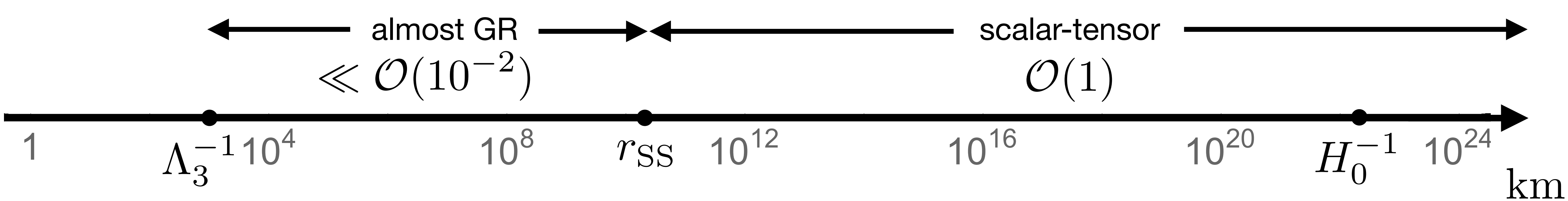
Scalar-tensor: $\mathcal{L} = f(\phi)R + G_2(\phi, X)$

Braiding: $\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$

$\frac{\Box\phi}{\Lambda_3^3} \gg 1$ Vainshtein screening: large classical scalar field nonlinearities



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Simplest models of modified gravity are base on single scalar field (universal coupling)

Quintessence: $$\mathcal{L} = R + V(\phi) - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

k-essence: $$\mathcal{L} = R + G_2(\phi, X)\ , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Scalar-tensor: $$\mathcal{L} = f(\phi)R + G_2(\phi, X)$$

Braiding: $$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \qquad \Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$$

Horndeski - most general Lorentz-invariant theory with up to 2nd-order EOM:

$$\begin{aligned}\mathcal{L} = {} & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \\ & - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]\end{aligned}$$

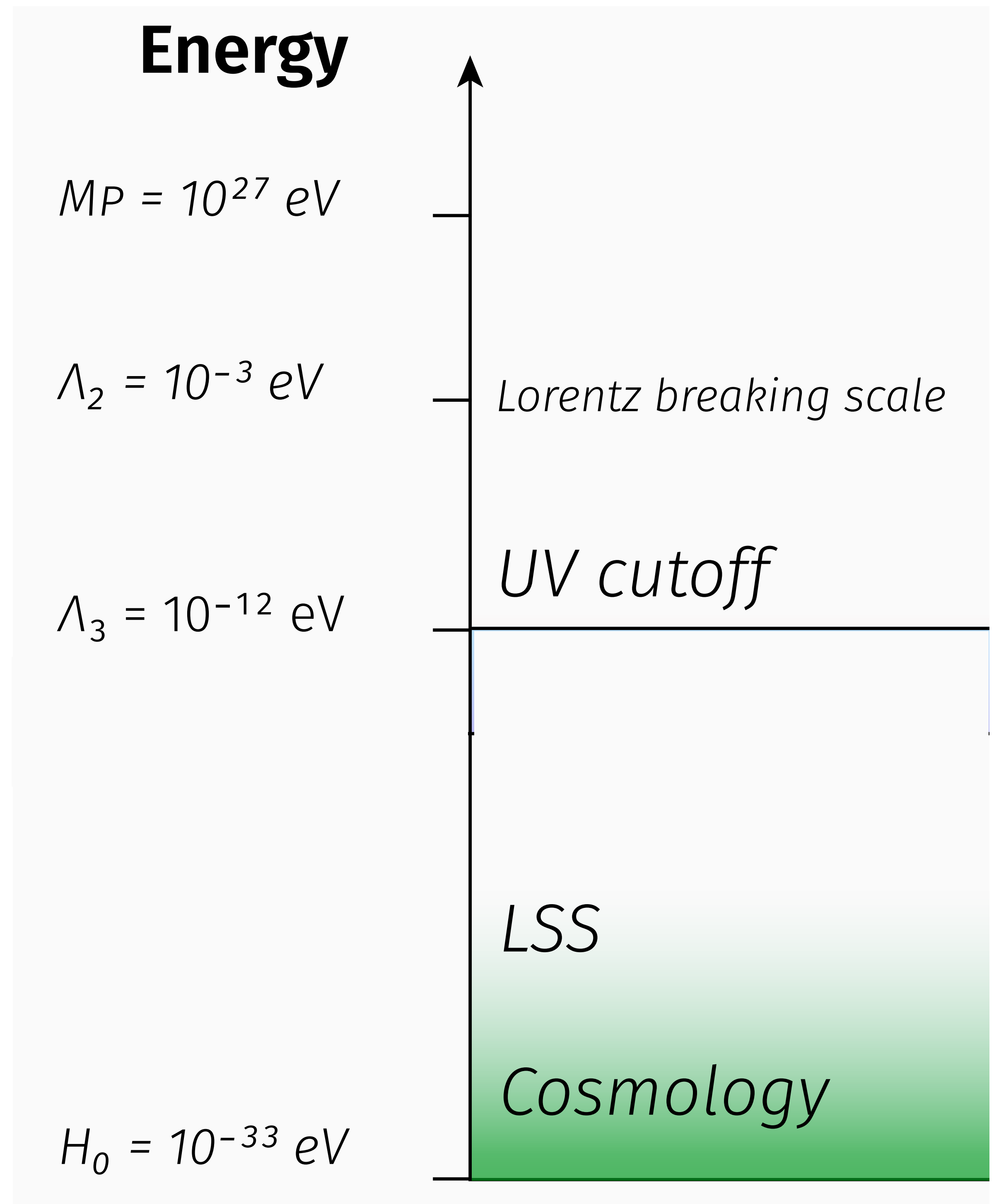
EFT expansion

Leading terms in a derivative expansion, relevant for cosmology. No Ostrogradski ghost guaranteed by 2nd order EOM

$$X\left(\frac{X}{\Lambda_2^4}\right)^n\left(\frac{\partial^2\phi}{\Lambda_3^3}\right)^m \qquad m=0,\,1,\,2,\,3$$

$$\Lambda_2\sim(M_{\rm Pl}H_0)^{1/2}\sim(\mathrm{mm})^{-1}$$

$$\Lambda_3\sim(M_{\rm Pl}H_0^2)^{1/3}\sim(1000\ \mathrm{km})^{-1}$$



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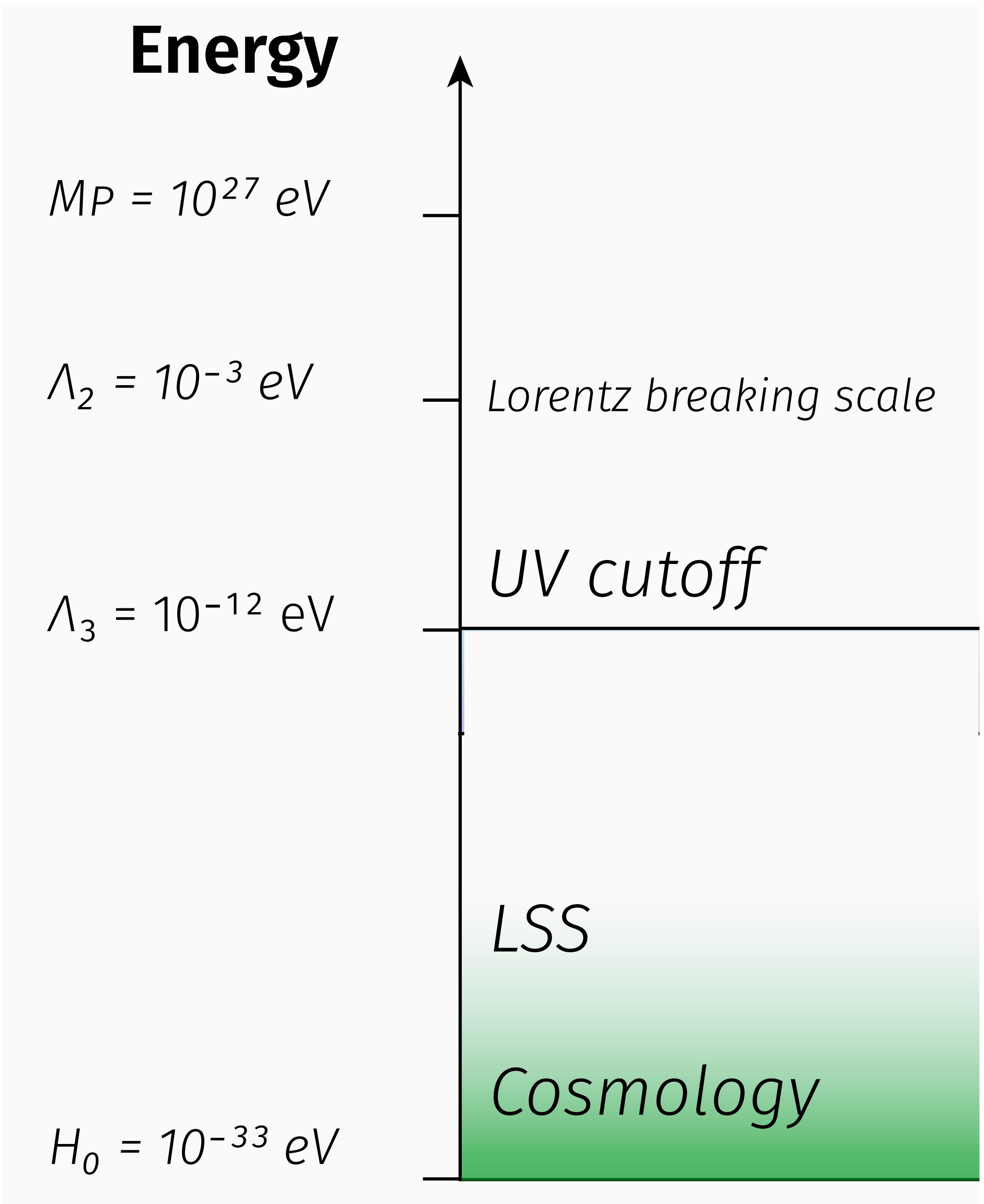
$$\Lambda_3 \sim (M_{\rm Pl}H_0^2)^{1/3} \sim (1000\ \mathrm{km})^{-1}$$

Higher-derivative operators relevant at Λ_3:

$$\Lambda_3^4\left(\frac{X}{\Lambda_2^4}\right)^n\left(\frac{\partial}{\Lambda_3}\right)^p\left(\frac{\partial^2\phi}{\Lambda_3^3}\right)^q$$

Approximate Galileon invariance protects from quantum corrections $\quad \phi \to \phi + c + b_\mu x^\mu$

$$\delta\lambda_i \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40}$$



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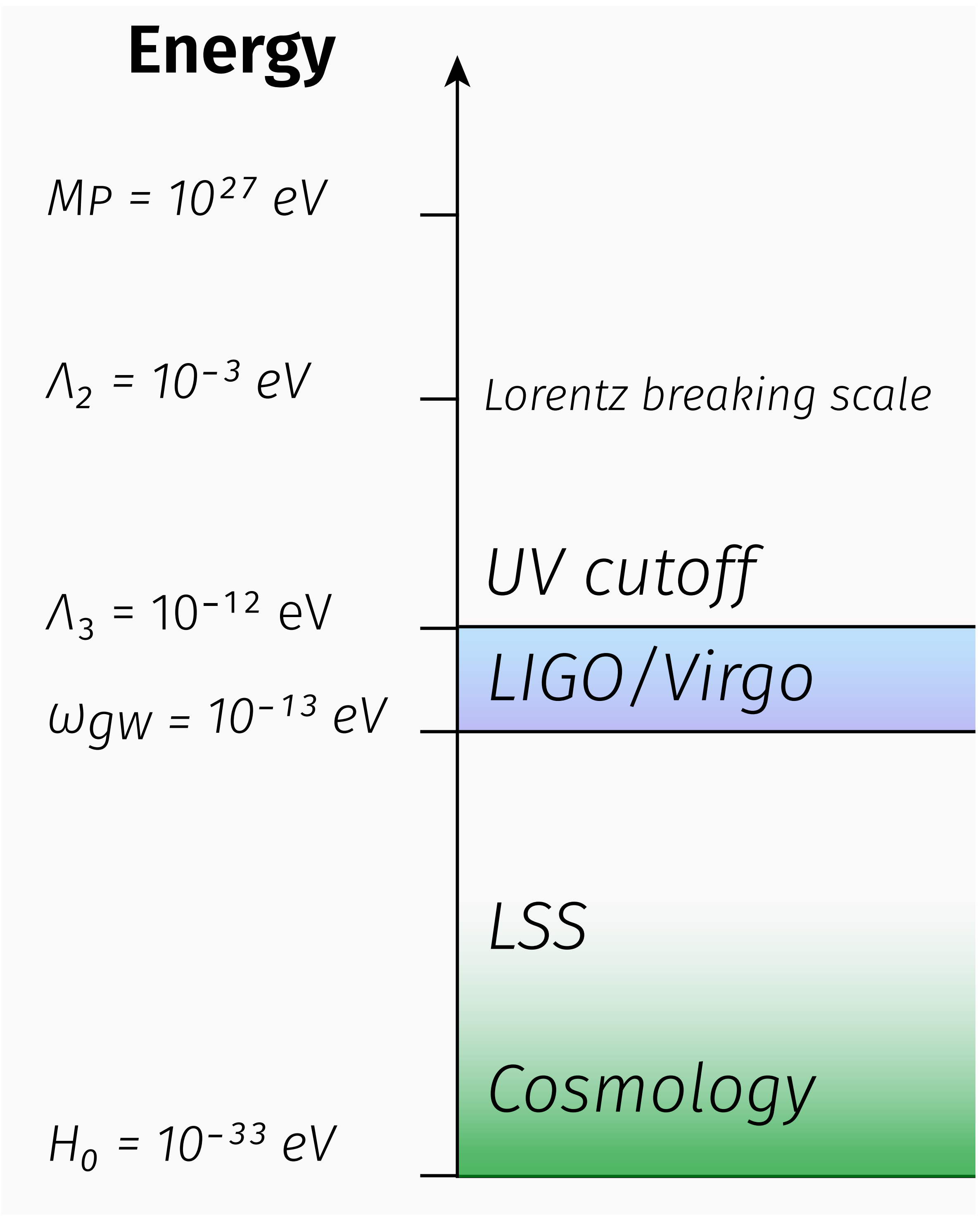

Beyond Horndeski

Horndeski

$$\mathcal{L} = G_4(\phi,X)R + G_2(\phi,X) + G_3(\phi,X)\Box\phi$$
$$- 2G_{4,X}(\phi,X)\Big[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\Big]$$
$$+ G_5(\phi,X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}G_{5,X}(\phi,X)\Big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\Big]$$

Invariance: $\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$

Beyond Horndeski

Horndeski

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$
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Invariance: $\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$

$D(X)$

GLPV

Horndeski + $F_{4,5}(\phi, X)$

Invariance: $\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$

In vacuum, two disformally related theories are equivalent. With matter (and minimal coupling), two *D*(*X*)-disformally related theories are inequivalent.

Beyond Horndeski

Horndeski

$$
\begin{aligned}
\mathcal{L} = {} & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \\
& - 2G_{4,X}(\phi, X)\Big[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\Big] \\
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\end{aligned}
$$

Invariance: $\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$

$D(X)$

GLPV

Horndeski + $F_{4,5}(\phi, X)$

Invariance: $\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$

$C(X)$

DHOST/EST

6 functions of ϕ, X

Invariance: $\tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$

EFT of Dark Energy

Bridge models and observations
in a minimal and systematic way

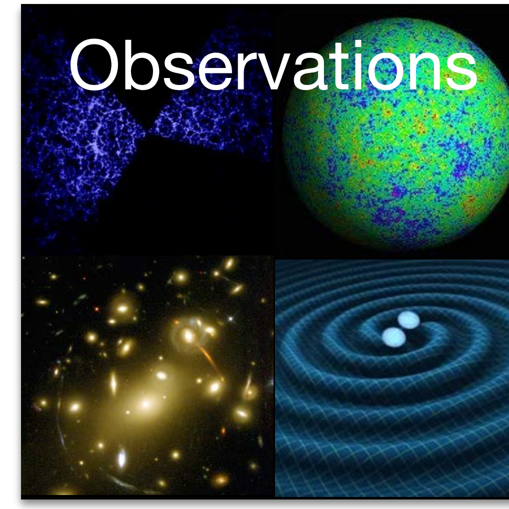
Space of theories

$$
\begin{aligned}
&G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \\
&- 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right] \\
&+ G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X) \\
&\times \left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\
&+ \dots
\end{aligned}
$$

EFT of DE

$$\alpha_1(t),\ \alpha_2(t),\ \dots$$

Observations



EFT of Dark Energy

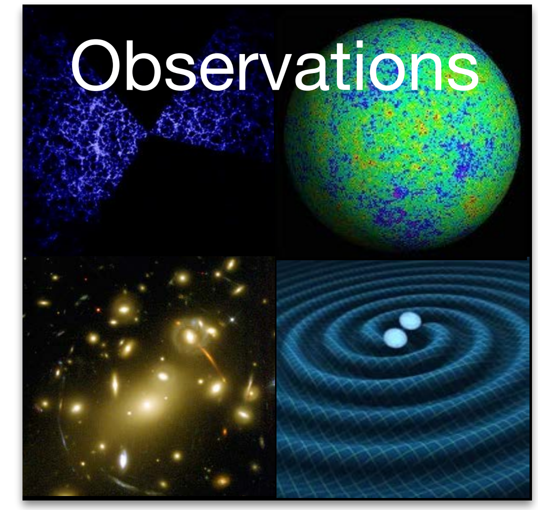
Bridge models and observations
in a minimal and systematic way

Space of theories

$$G_4(\phi,X)R + G_2(\phi,X) + G_3(\phi,X)\Box\phi$$
$$- 2G_{4,X}(\phi,X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right]$$
$$+ G_5(\phi,X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi,X)$$
$$\times\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]$$
$$+\dots$$

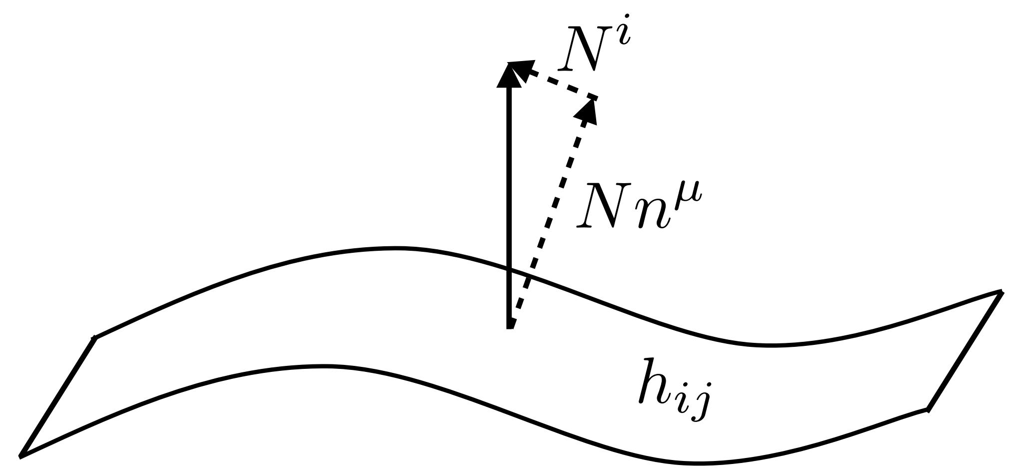
EFT of DE

$$\alpha_1(t),\ \alpha_2(t),\ \dots$$



Action contains all possible scalars under spatial diffs, ordered by number of perturbations and derivatives

$$ds^2 = -N^2dt^2 + h_{ij}(N^idt + dx^i)(N^jdt + dx^j)$$



$$S = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K^j_i\delta K^i_j - \delta K^2 + c_T^2\,{}^{(3)}R + \delta N\,{}^{(3)}R + \alpha_K\delta N^2 + 4H\alpha_B\delta K\delta N + \dots\right.$$

$$\alpha_M = \frac{d\ln M^2}{d\ln a}$$

Lapse	N	$\sim\dot\phi$	$(\partial\phi)^2 = -\dot\phi_0^2(t)/N^2$
Extrinsic curvature	K_{ij}	$\sim\partial_t g_{ij}$	$K_{ij} = \frac{1}{2N}(\dot h_{ij} - \nabla_iN_j - \nabla_jN_i)$
Intrinsic curvature	${}^{(3)}R_{ij}$	$\sim\partial^2 g_{ij}$	

EFT of Dark Energy

Bridge models and observations
in a minimal and systematic way

Space of theories

$$
\begin{aligned}
&G_4(\phi,X)R + G_2(\phi,X) + G_3(\phi,X)\Box\phi \\
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&+ G_5(\phi,X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi,X) \\
&\times \left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\
&+ \dots
\end{aligned}
$$

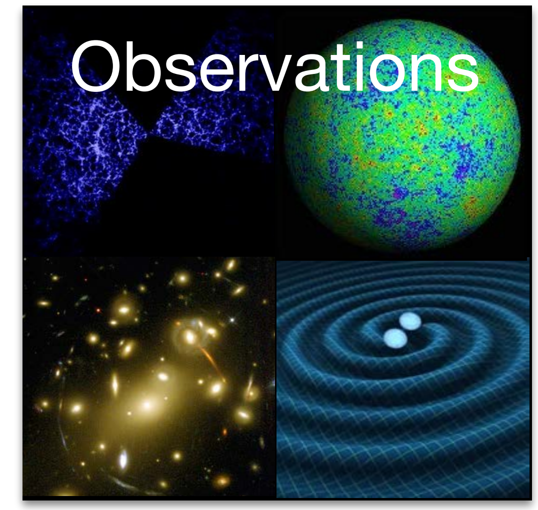
←

EFT of DE

$\alpha_1(t),\ \alpha_2(t),\ \dots$

→

Observations

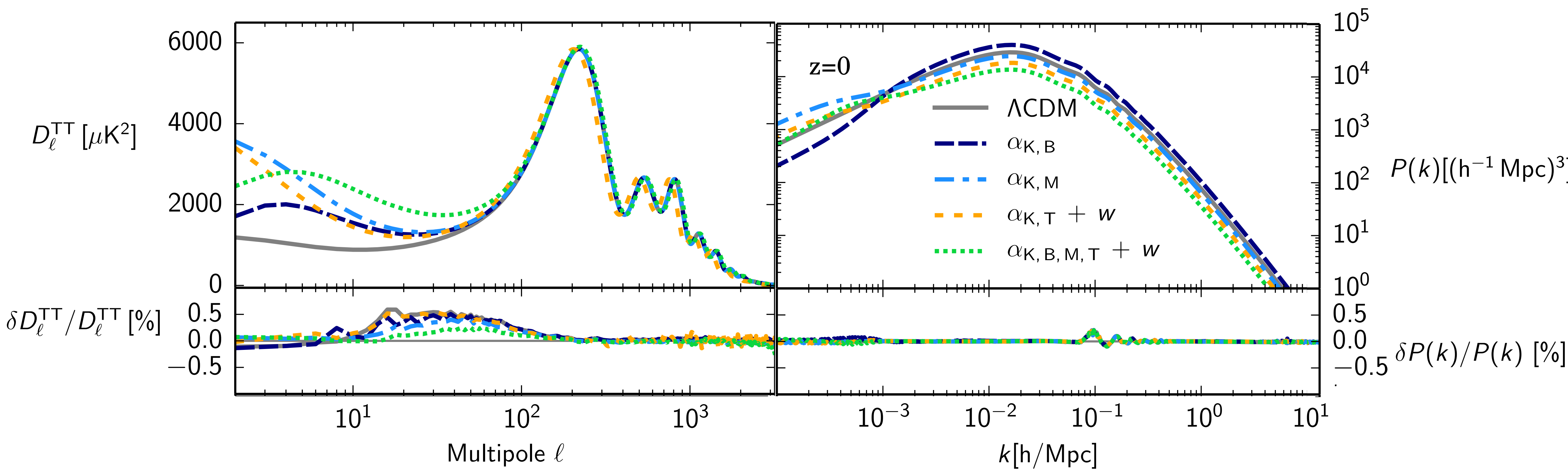


$$\mu = \mu\big(k; \alpha_1(t), \alpha_2(t), \dots\big)$$

$$\Sigma = \Sigma\big(k; \alpha_1(t), \alpha_2(t), \dots\big)$$

$$\nabla^2\Phi = 4\pi G\,\mu\,\delta\rho_{\rm m}$$

$$\nabla^2(\Phi + \Psi) = 8\pi G\,\Sigma\,\delta\rho_{\rm m}$$



EFT of Dark Energy

Linear perturbations: from CMB + large-scale structure $|\alpha_i| \lesssim 0.01 \div 0.1$

Horndeski	$\alpha_K(t), \quad \alpha_M(t), \quad \alpha_B(t), \quad c_T(t) - 1$
GLPV	$\alpha_H(t)$
DHOST/EST	$\beta_1(t)$

GW propagation

GW propagation in FLRW

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 0$$

$$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + \gamma_{ij}\right] d\vec{x}^i d\vec{x}^j \ , \qquad \gamma_{ii} = 0 = \partial_i \gamma_{ij} \ , \qquad H = \dot{a}/a$$

GW propagation

Lorentz breaking acts like a medium: gravitons are absorbed and dispersed.

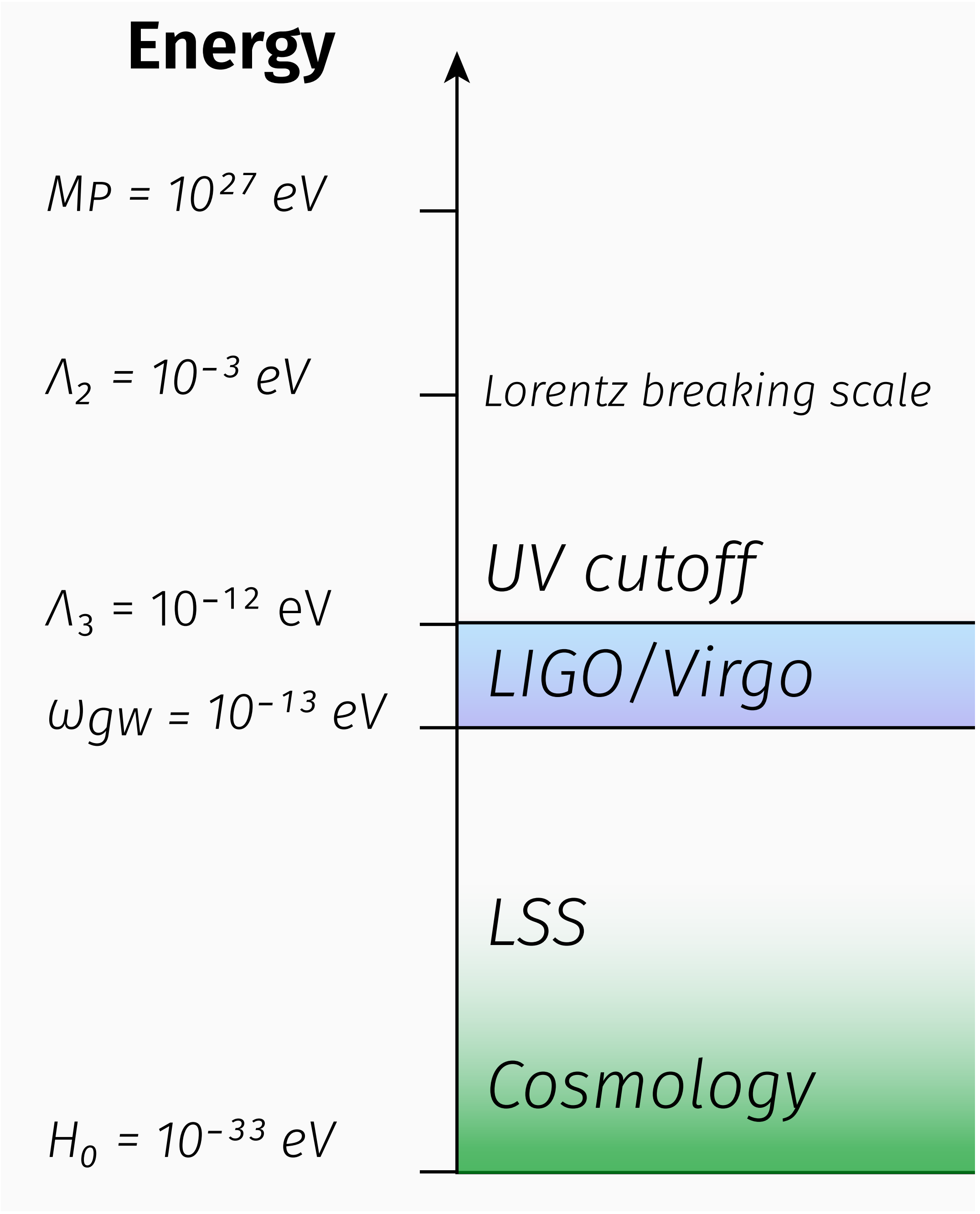
$$\ddot{\gamma}_{ij} + H(3 + \alpha_M)\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} = 0$$

- LISA: $|\alpha_M| < 0.03 \div 0.1$
- GW170817: $|c_T - 1| < 10^{-15}$

$$\delta c_T \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40} \ll 10^{-15}$$

tuning is stable!

$$\mu = \mu(\alpha_M, c_T^2, \ldots) \,, \qquad \Sigma = \Sigma(\alpha_M, c_T^2, \ldots)$$



GW constraints

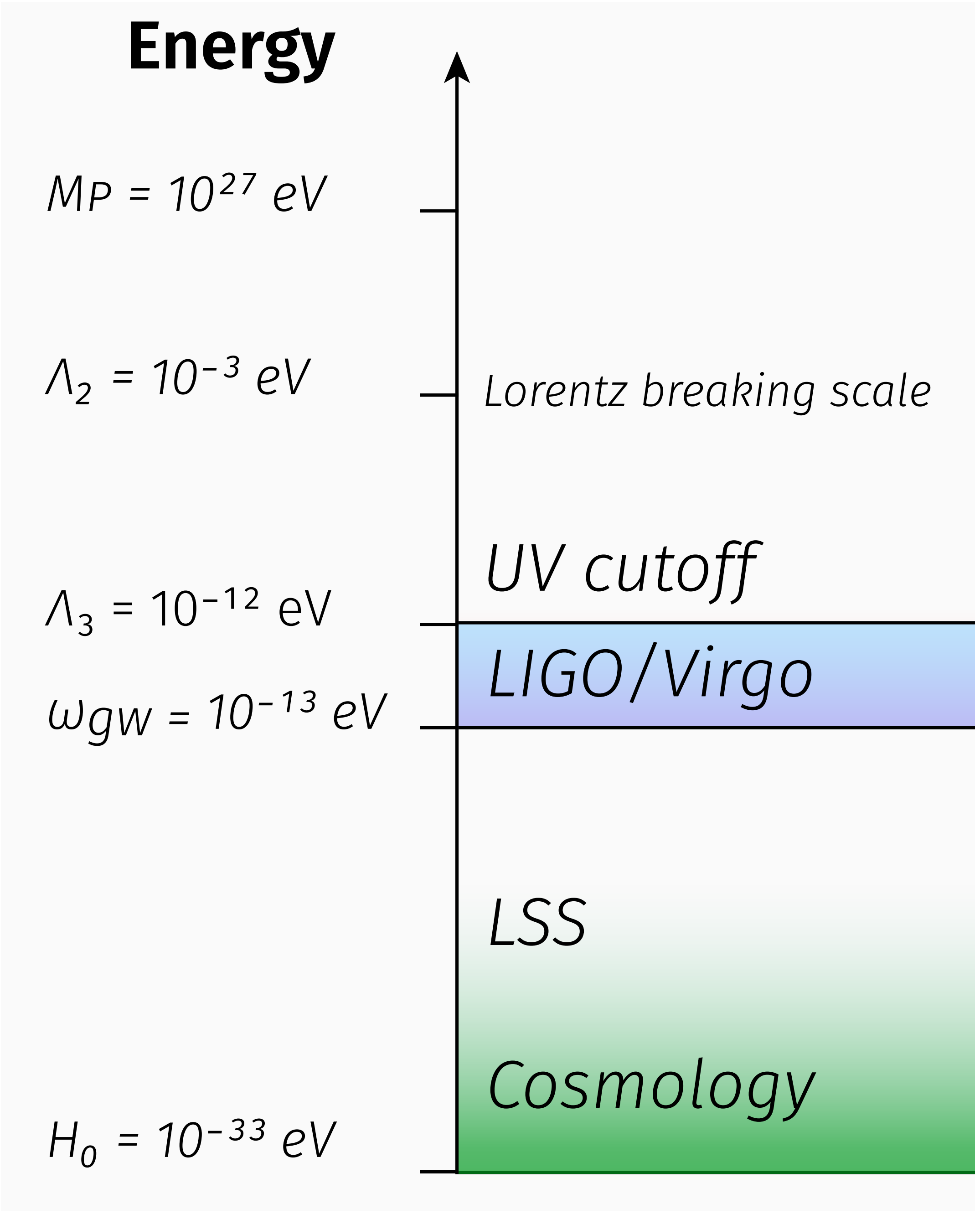
	$c_T=1$	$c_T \neq 1$
Horndeski	$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$ Linear and nonlinear (quasi-static limit) functions for $c_T=1$: $\alpha_K(t)\ ,\ \alpha_M(t)\ ,\ \alpha_B(t)$	$\alpha_{\rm V1} = \alpha_{\rm V2} = \alpha_{\rm V3} = 0$
GLPV	$\alpha_H(t)$	
DHOST/EST	$\beta_1(t)$	

GW propagation

Lorentz breaking acts like a medium: gravitons are absorbed and dispersed.

$$\ddot{\gamma}_{ij} + [(3+\alpha_{\rm M})H + \Gamma(k)]\,\dot{\gamma}_{ij} + \left[c_T^2 k^2 + f(k)\right]\gamma_{ij} = 0$$

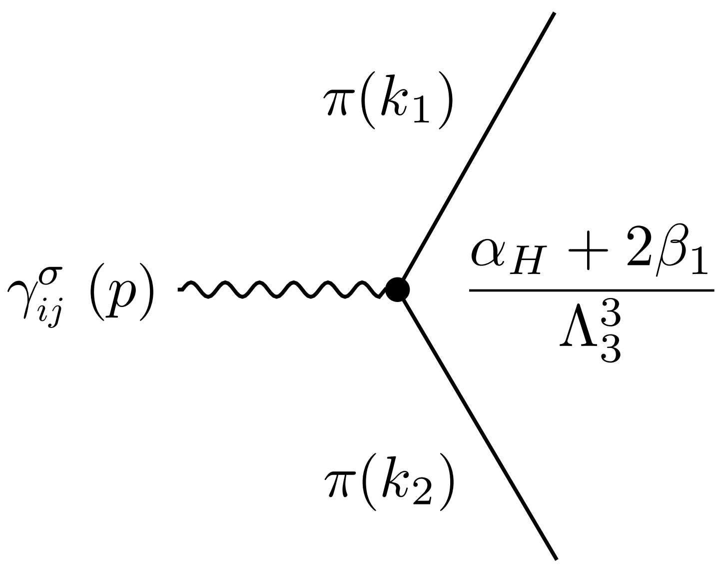
- LISA: $|\alpha_M| < 0.03 \div 0.1$
- GW170817: $|c_T - 1| < 10^{-15}$
- $d_S \Gamma \ll 1 \quad \Rightarrow \quad |\alpha_H + 2\beta_1| \ll 10^{-8}$



$$\mu = \mu(\alpha_M, c_T^2, \dots)\ , \qquad \Sigma = \Sigma(\alpha_M, c_T^2, \dots)$$

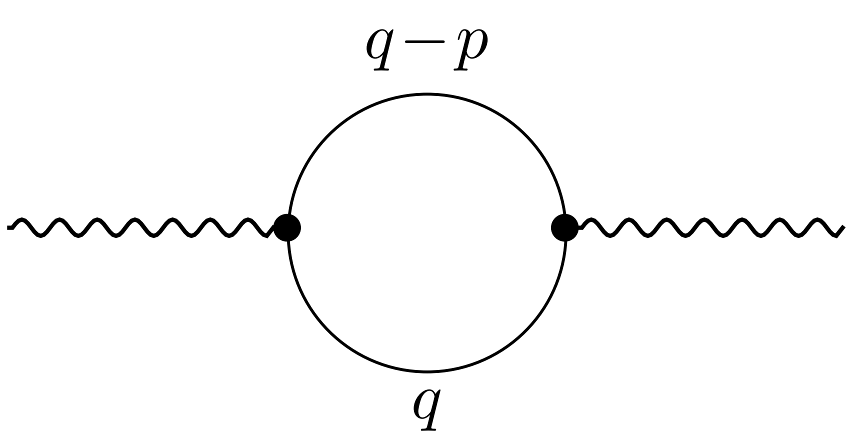
The decay of GW

Cubic interactions between GW and scalar fluctuations $\pi \equiv \delta\phi/\dot{\phi}_0$: decay of GWs for $c_s < 1$
(c_s = sound speed of π fluctuations)



$$\Gamma \simeq \left(\frac{\alpha_H + 2\beta_1}{\Lambda_3^3}\right)^2 \frac{k^7(1-c_s^2)^2}{c_s^7}$$

Graviton self-energy (valid also for $c_s > 1$):



$$\omega = k^2 - \left(\frac{\alpha_H + 2\beta_1}{\Lambda_3^3}\right)^2 \frac{k^8(1-c_s^2)^2}{\pi c_s^7} \log\left((c_s^2 - 1)\frac{k^2}{\mu^2}\right)$$

Optical theorem: $\Gamma(k)\omega(k) = \mathrm{Im}\left[f(k)\right]$

GW propagation

Lorentz breaking acts like a medium: gravitons are absorbed and dispersed.

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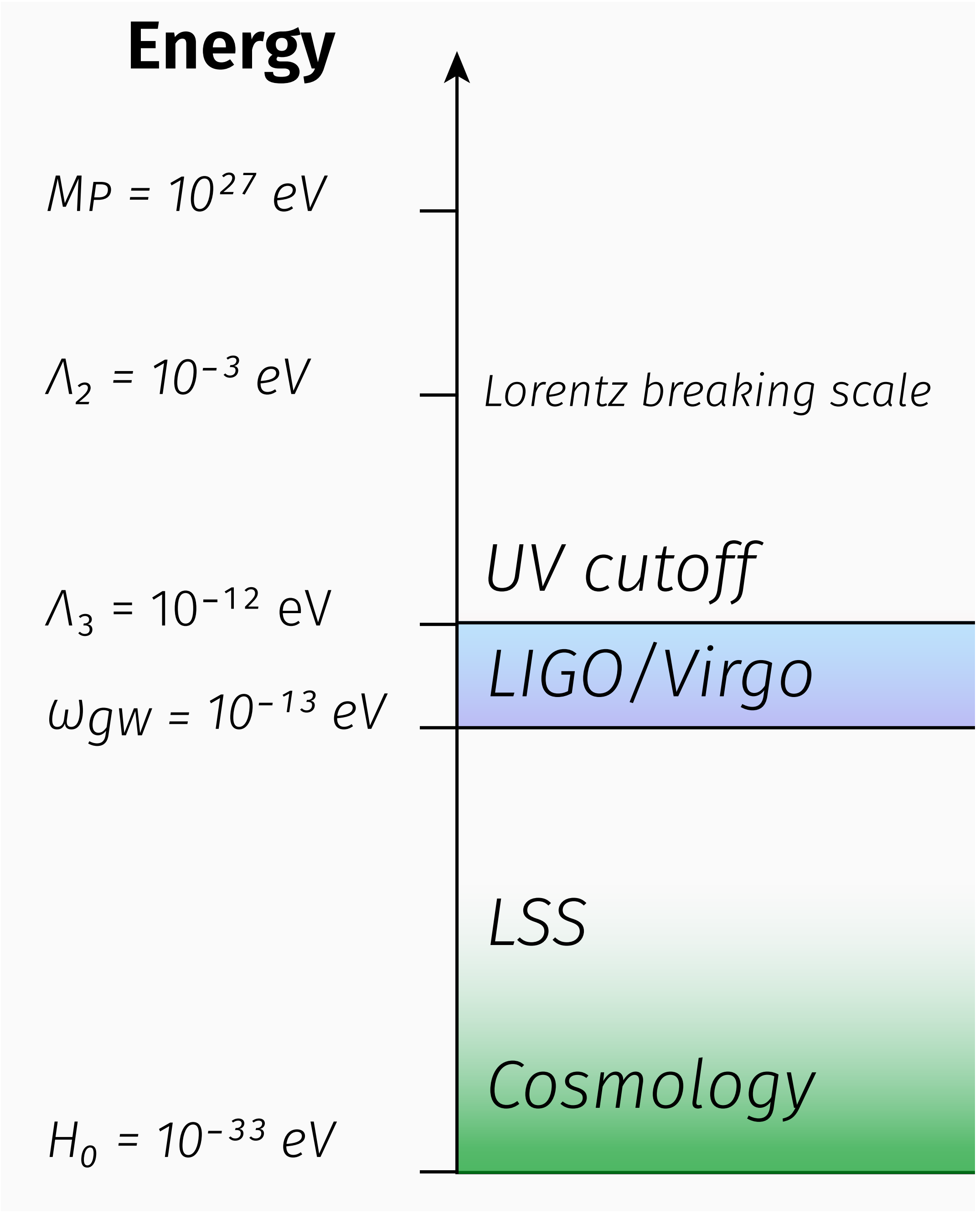

$$\mu = \mu(\alpha_M, c_T^2, \dots)\,, \qquad \Sigma = \Sigma(\alpha_M, c_T^2, \dots)$$

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GLPV		
DHOST/EST	No decay $\alpha_H(t) = -2\beta_1(t)$	

GW propagation

Lorentz breaking acts like a medium: gravitons are absorbed and dispersed.

$$\ddot{\gamma}_{ij} + [(3+\alpha_{\rm M})H + \Gamma(k)]\,\dot{\gamma}_{ij} + \left[c_T^2 k^2 + f(k)\right]\gamma_{ij} = 0$$

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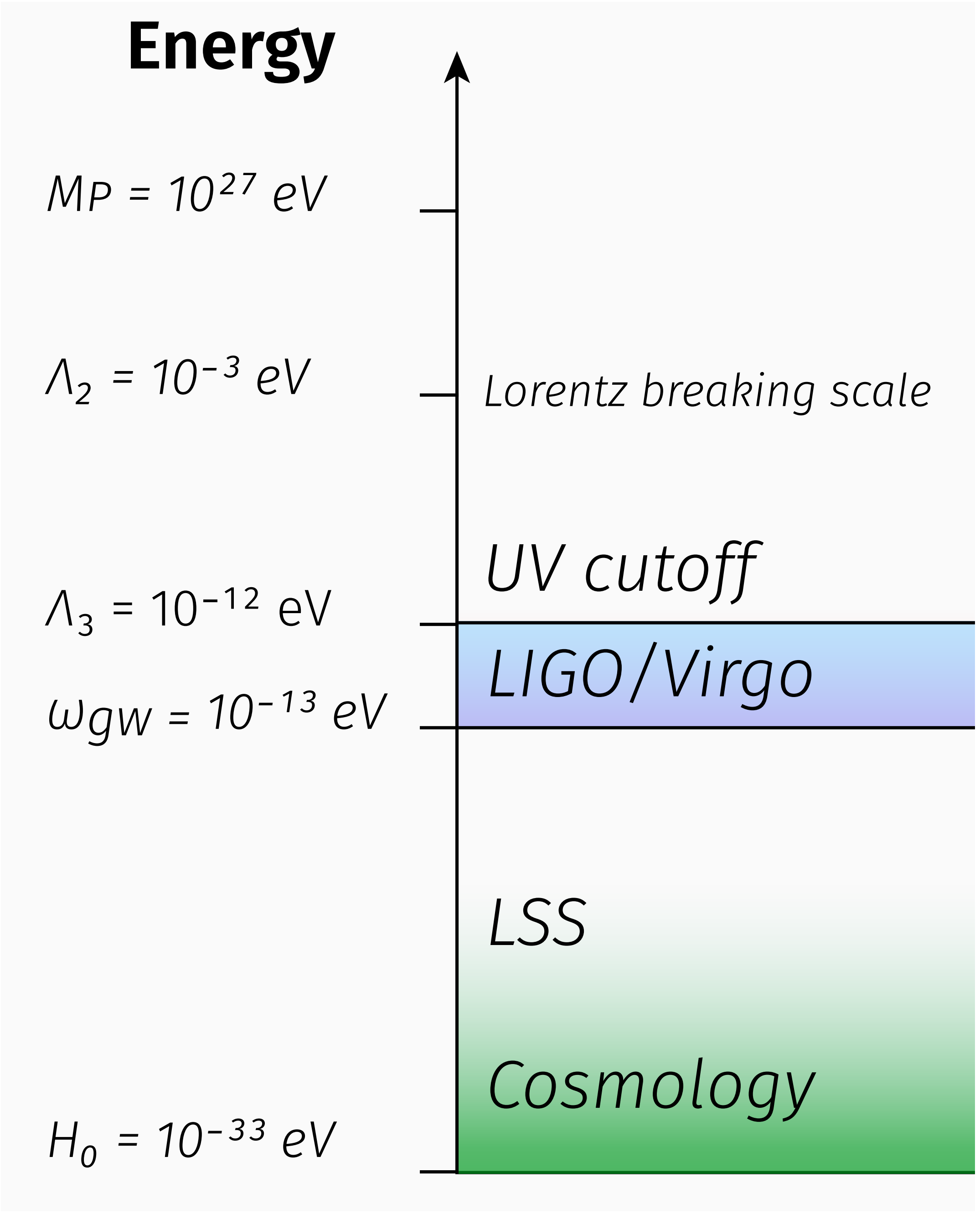
- $d_S \Gamma \ll 1 \quad \Rightarrow \quad |\alpha_H + 2\beta_1| \ll 10^{-8}$

- Can we constrain other operators with the decay?
- Can we bypass these constraints?

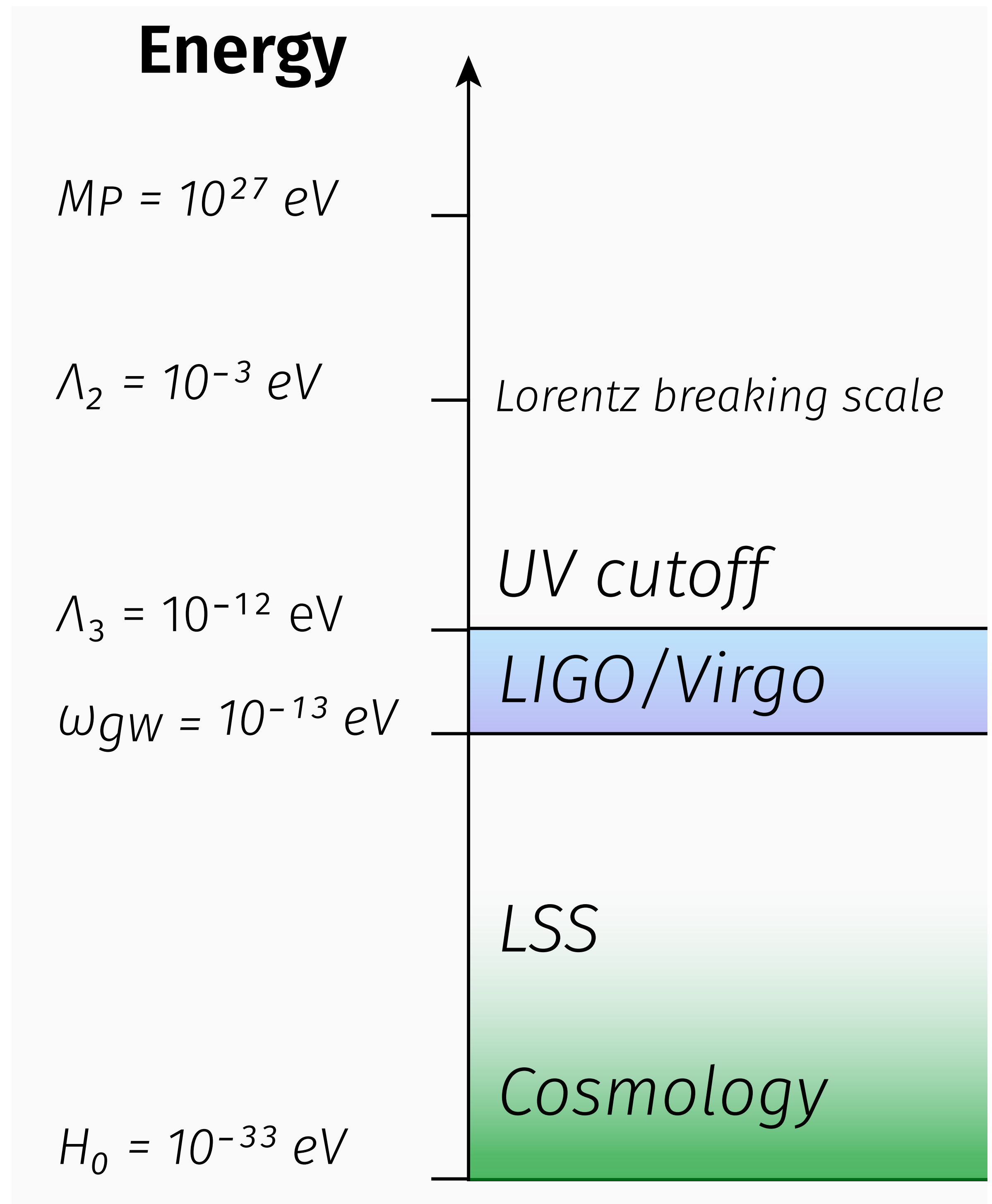
$$\mu = \mu(\alpha_M, c_T^2, \ldots)\ , \qquad \Sigma = \Sigma(\alpha_M, c_T^2, \ldots)$$



Caveat: Lorentz inv. in the UV

Lorentz invariance could be recovered in the UV, before unitarity cutoff Λ_3

$$\omega^2 = c_T^2 k^2 + \frac{k^4}{M^2} + \dots$$
$$= k^2 \left(1 + \mathcal{O}(M^2/k^2)\right) \qquad M \ll \Lambda_3$$



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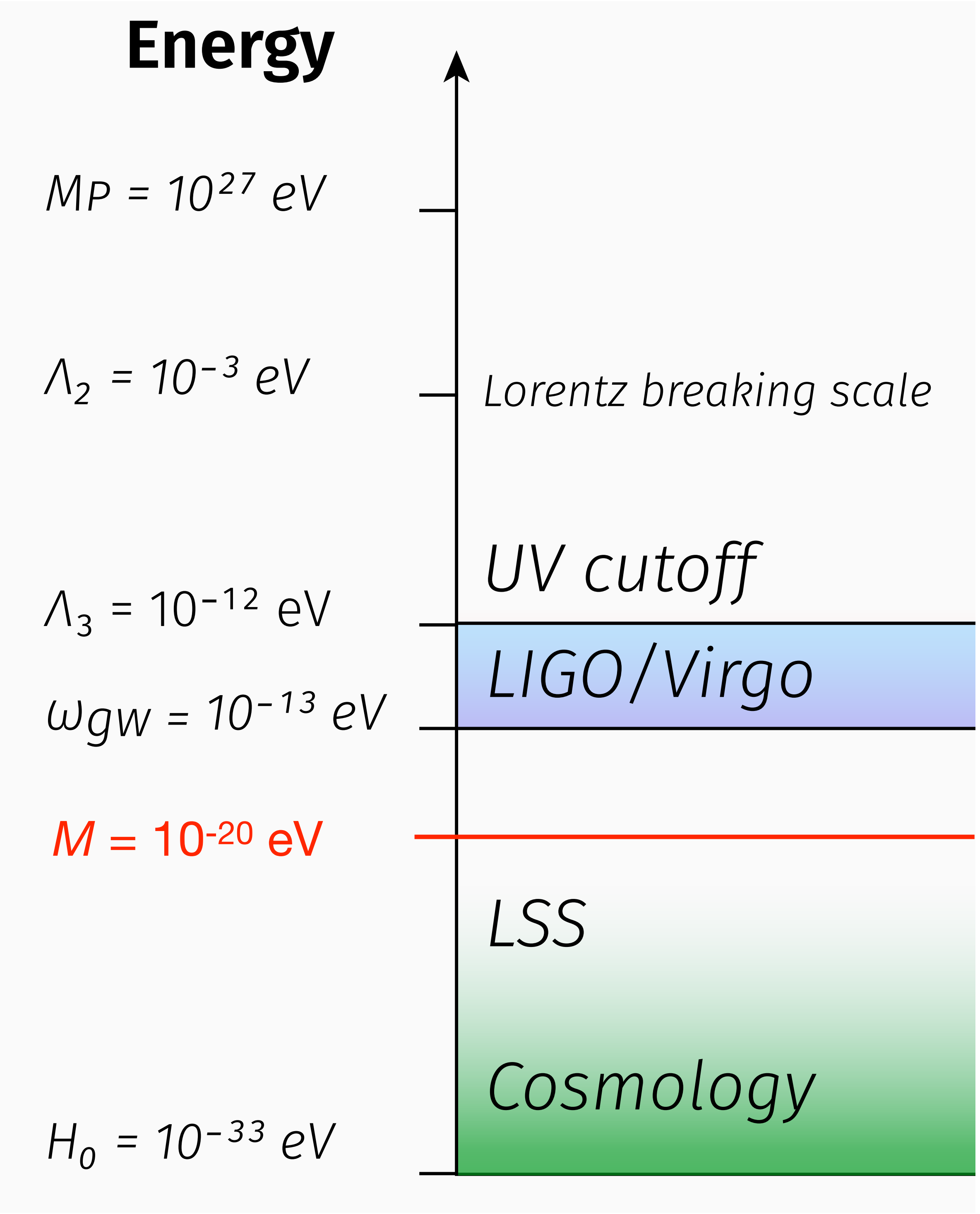
$$= k^2 \left(1 + \mathcal{O}(M^2/k^2)\right) \qquad M \ll \Lambda_3$$

Scale of new degrees of freedom *M* is parametrically smaller than Λ_3. Naively:

$$M \lesssim 10^{-8}\Lambda_3 \sim (10^{11}\ \mathrm{km})^{-1}$$

- How to reconcile with local tests of gravity?
- Can we say something general about UV completion? Analogous to frequency dependent refraction index: Kramers-Kronig?



Part 1: Lorentz violations
Part 2: Alternative theories as strong field parametrizations

Thomas P. Sotiriou

EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Science & Technology Facilities Council

The University of Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

Motivation for LV

Test Lorentz symmetry!

Theories

Einstein-aether theory

Field content: metric $g_{\mu\nu}$, aether u^μ (preferred 'threading')

Dispersion: Linear $\omega \propto k$

UV completion: unknown

T. Jacobson and D. Mattingly, PRD 64, 024028 (2001).

Hořava gravity

Field content: metric $g_{\mu\nu}$, scalar T (preferred foliation)

Dispersion: non-linear $\omega^2 \propto k^2 + ak^4 + ...$

UV completion: known

P. Hořava, PRD 79, 084008 (2009)
D. Blas, O. Pujolas and S. Sibiryakov, PRL 104, 181302 (2010)

Propagation effects

$$E^2 = m_g^2 \pm M_1 p + c_g^2 p^2 \pm \frac{p^3}{M_3} \pm \frac{p^4}{M_4^2} + \dots$$

- Strong bound on the mass of the graviton, M_1, M_3
- But marginally interesting from a theory perspective
- Weak bounds on M_4 in eV range
- Strong constraint from BNS and EM

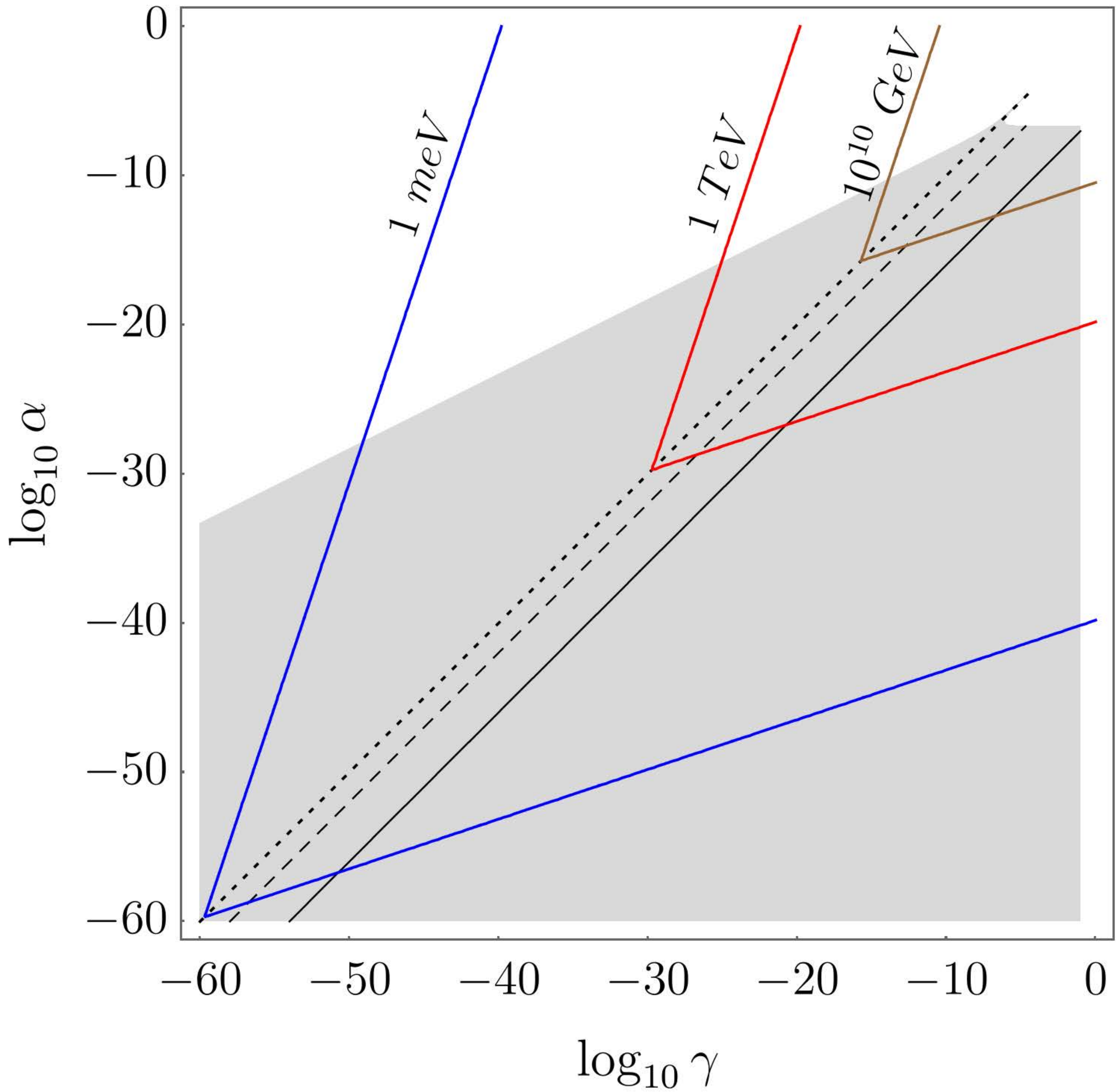
$$|\Delta c_g/c| \lesssim 10^{-15}$$

But we can do better in constraining Lorentz violations by looking for other polarisations!

T.P.S., PRL 120, 041104 (2018);

Combined Constraints

Hořava gravity



A. E. Gumrukcuoglu, M. Saravani and T.P.S., PRD 97, 024032 (2018)

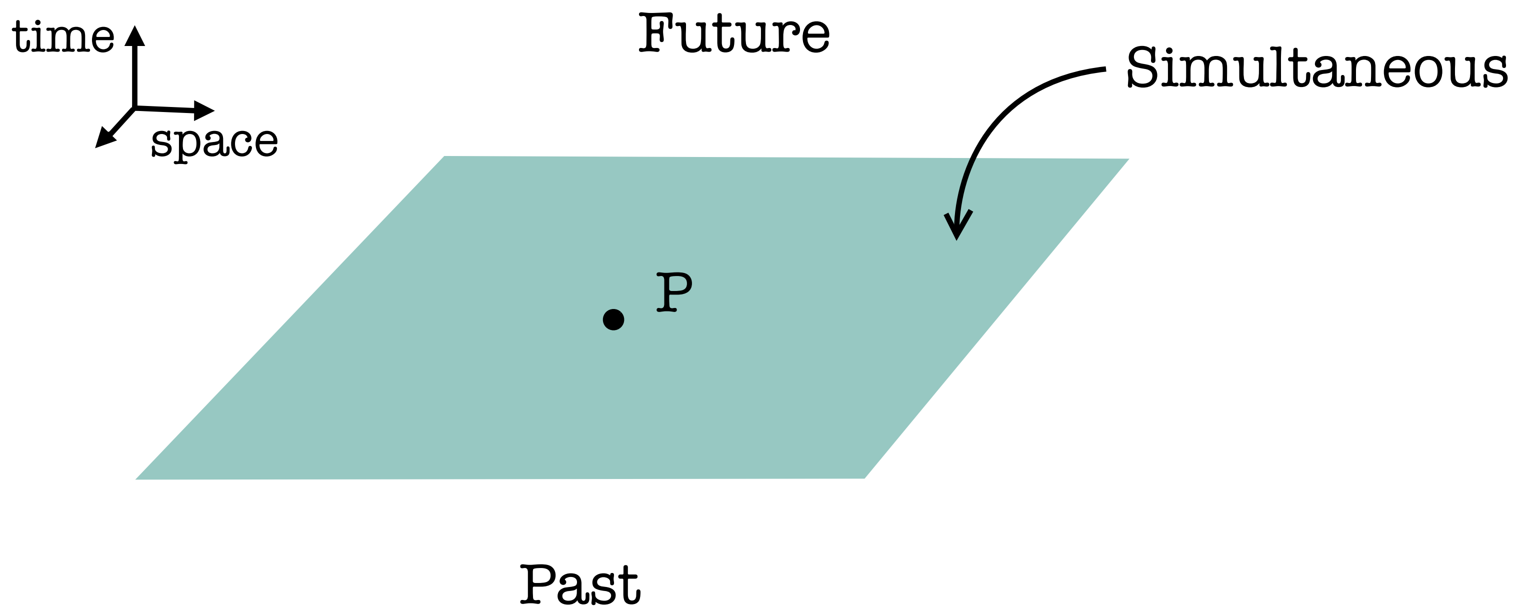
Black holes

LV with linear dispersion relations $\omega \propto k$: effective metrics

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + (s_i^2 - 1) u_\mu u_\nu$$

…and multiple horizons.

LV with non-linear dispersion relations $\omega^2 \propto k^2 + ak^4 + \ldots$



…'universal horizons'.

E. Barausse, T. Jacobson and T.P.S., PRD 83, 124043 (2011)
D. Blas and S. Sibiryakov, PRD 84, 124043 (2011)
M. Colombo, J. Bhattacharyya, and T.P.S., CQG 33, 235003 (2016).

Black holes

- The universal horizons is always behind Killing horizons

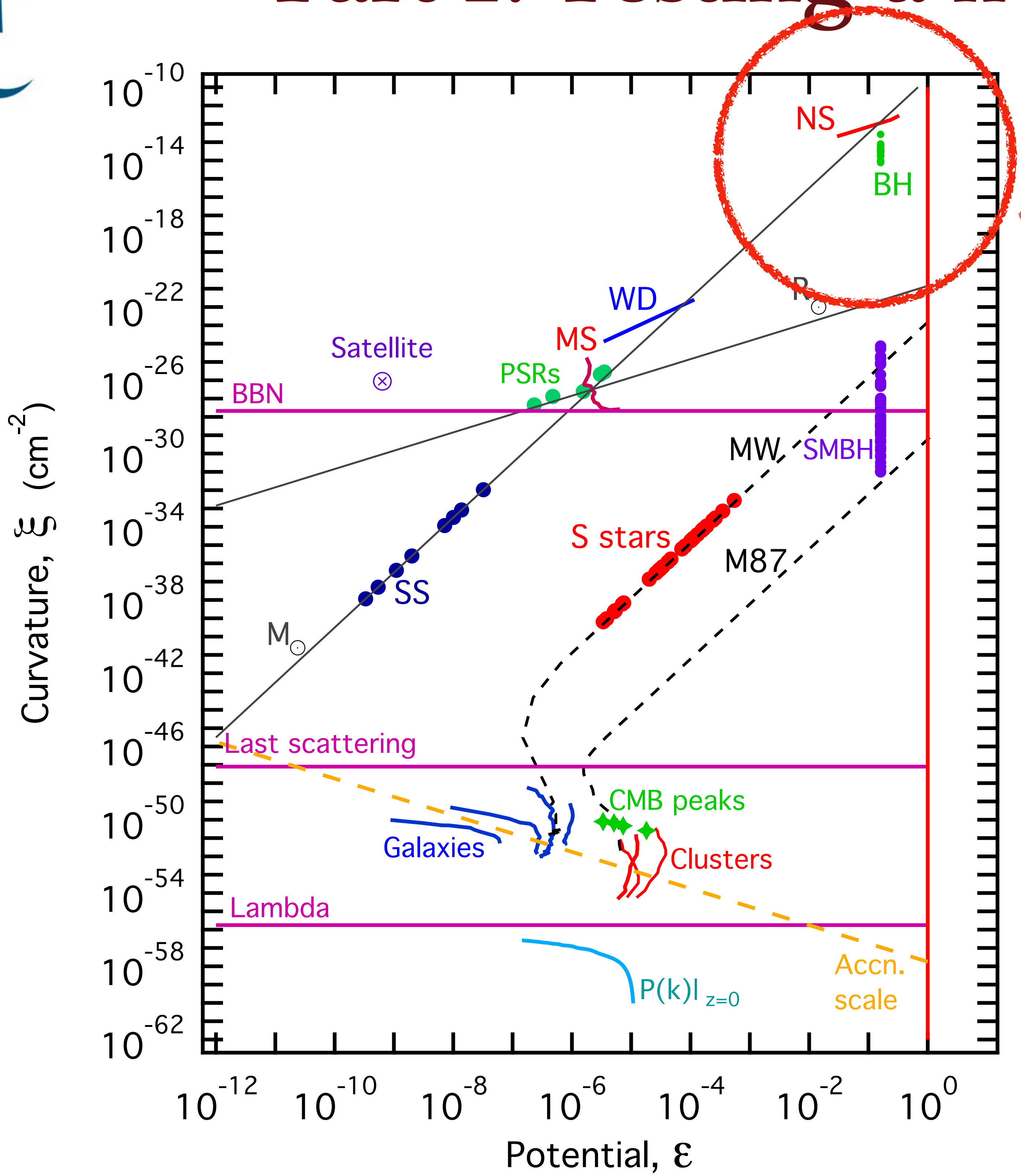
M. Colombo, J. Bhattacharyya, and T.P.S., CQG 33, 235003 (2016).

- The exterior is always 'hairy' - deviations from GR always present due to LV

- The deviations are (probably) at percent level in the allowed part of the parameter space

E. Barausse, T. Jacobson and T.P.S., PRD 83, 124043 (2011)
E. Barausse and T.P.S., Phys. Rev. Lett. 109, 181101 (2012)
E. Barausse and T.P.S., Phys. Rev. D 87, 087504 (2013)
E. Barausse, I. Vega and T.P.S., Phys. Rev. D 93, 044044 (2016)

Part 2: Testing a new regime



Untested combination of curvature and potential

taken from T. Baker, D. Psaltis, C. Skordis, ApJ 802, 63 (2015)

Motivation

For a given observed system, what kind of theory could exhibit new effects, while consistent with other observations?

Pinning down the theory

Assume that

- system of interest: black holes
- field: massless scalar

No mass requires shift symmetry. No-hair theorem!

S.W. Hawking, Comm. Math. Phys. 25, 152 (1972)
L. Hui, A. Nicolis, PRL 110, 241104 (2013).

…but there is also a unique exception.

T.P.S. and S.-Y. Zhou, PRL 112, 251102 (2014)

$$S = \frac{m_P^2}{8\pi}\int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha\phi\mathcal{G}\right)$$

The exception

The corresponding scalar equation is

$$\Box\phi + \alpha\mathcal{G} = 0$$

- All black holes are hairy
- At small coupling/weak field identical to exponential coupling of dilaton in string theory

  P. Kanti et al., PRD 54, 5049 (1996)
  N. Yunes and L. Stein, PRD 83, 104002 (2011)

- First dynamical simulations done

  R. Benkel, T.P.S. and H. Witek, Phys. Rev. D 94 (R), 121503 (2016); Class. Quant. Grav. 34, 064001 (2017)
  H. Witek, L. Gualtieri, P. Pani and T.P.S., arXiv:1810.05177

- Neutron stars have no scalar monopole!

  K.Yagi, L. Stein and N. Yunes, PRD 93, 024010 (2016)

Spontaneous scalarization

In spherical symmetry + Einstein frame

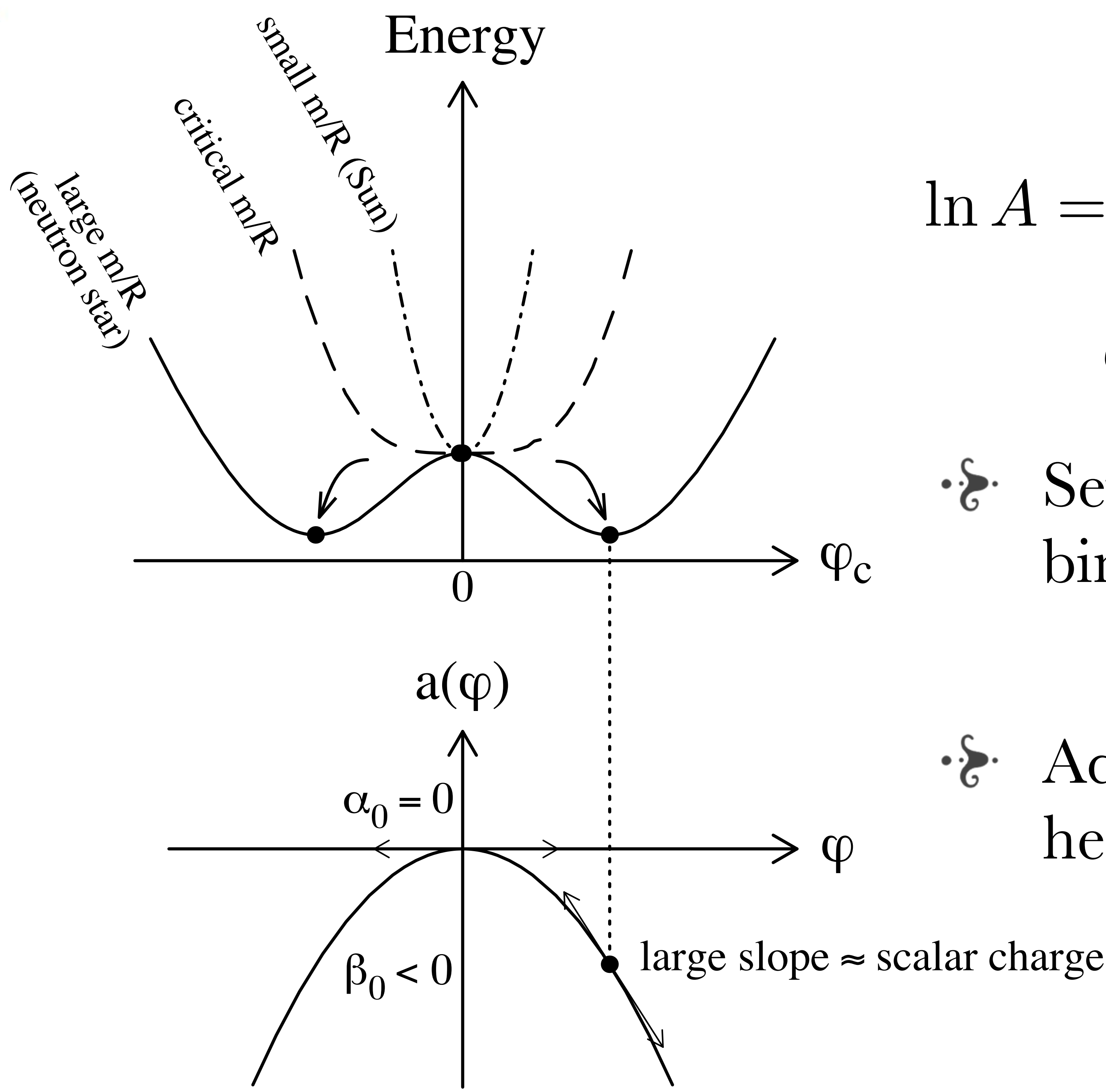
$$\Box\phi = A^3 A' T = U'_{\text{eff}}$$

- If $A'(\phi_0) = 0$ then the theory will admit GR solutions around matter!
- However they will not necessarily be the only ones...
- The non-GR configuration is preferred for sufficiently large central density

T. Damour and G. Esposito-Farese, Phys. Rev. Lett. 70, 2220 (1993)

Celebrated demonstration that strong field effects can be very important...

Spontaneous scalarization



$$\ln A = \beta_0(\phi - \phi_0)^2 + \ldots$$

$$\omega(\phi_0) \to \infty$$

- Severely constrained by binary pulsar tests!
- Adding a mass term helps

Taken from G. Esposito-Farese, arXiv:gr-cq/0402007

Black hole scalarization

H. O. Silva, J. Sakstein, L. Gualtieri, T.P.S, and E. Berti, PRL 120, 131104 (2018)

No-hair theorem for the action

$$S = \frac{m_P^2}{8\pi}\int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + f(\phi)\mathcal{G}\right)$$

provided that $\quad f'(\phi_0) = 0\,, \quad f''(\phi_0)\mathcal{G} < 0$

That is, for the equation

$$\Box\phi = -f'(\phi)\mathcal{G}$$

trivial solutions are unique if admissible, if the effective mass is positive

- But if it is negative then there can be "scalarization"!

See also: D. D. Doneva and S. S. Yazadjiev, PRL 120, 131103 (2018)

What we DO know

- How to consistently parameterize UV corrections to gravitational physics for Lorentz invariant UV completions
- How to build theories of gravity with massless and massive states, both hard (poles) and soft (resonances)
- How to construct general EFT for dark energy and cosmological predictions (at linear scales) and to constrain them with GW
- How to build EFTs for Lorentz symmetry breaking and some UV completions
- We know how to screen, and how usual GR tests are bypassed
- How to select theories with interesting strong field phenomenology

What we DON'T know

- (Asymptotically flat) Black Hole solutions in massive gravity/bigravity/multi-gravity
- How the quasi-normal mode spectrum is modified, how mergers are modified, realistic black holes, neutron stars
- How do we compute radiation/gravitational waveform for Vainshtein theories
- UV completion of Galileons, Horndeski, Soft and Hard Massive Gravity and general EFTs?
- Are there ways out from GW170817? (avoiding constraints on sound speed)
- Well-posedness in the nonlinear region?
- Strong equivalence principle violations in nonlinear region?