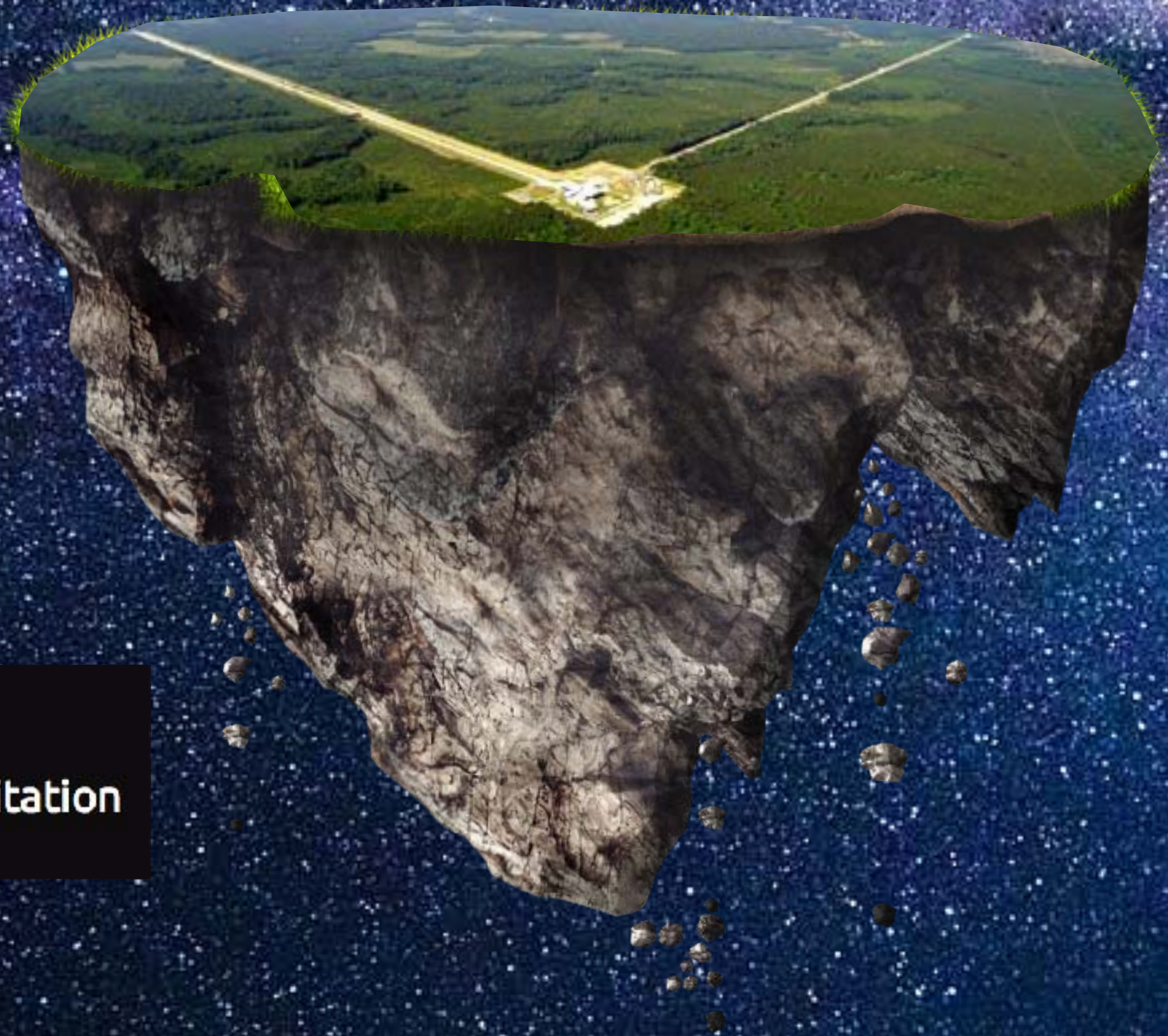
LISA Science Analysis Tools and Strategies- LISA is not LIGO in Space



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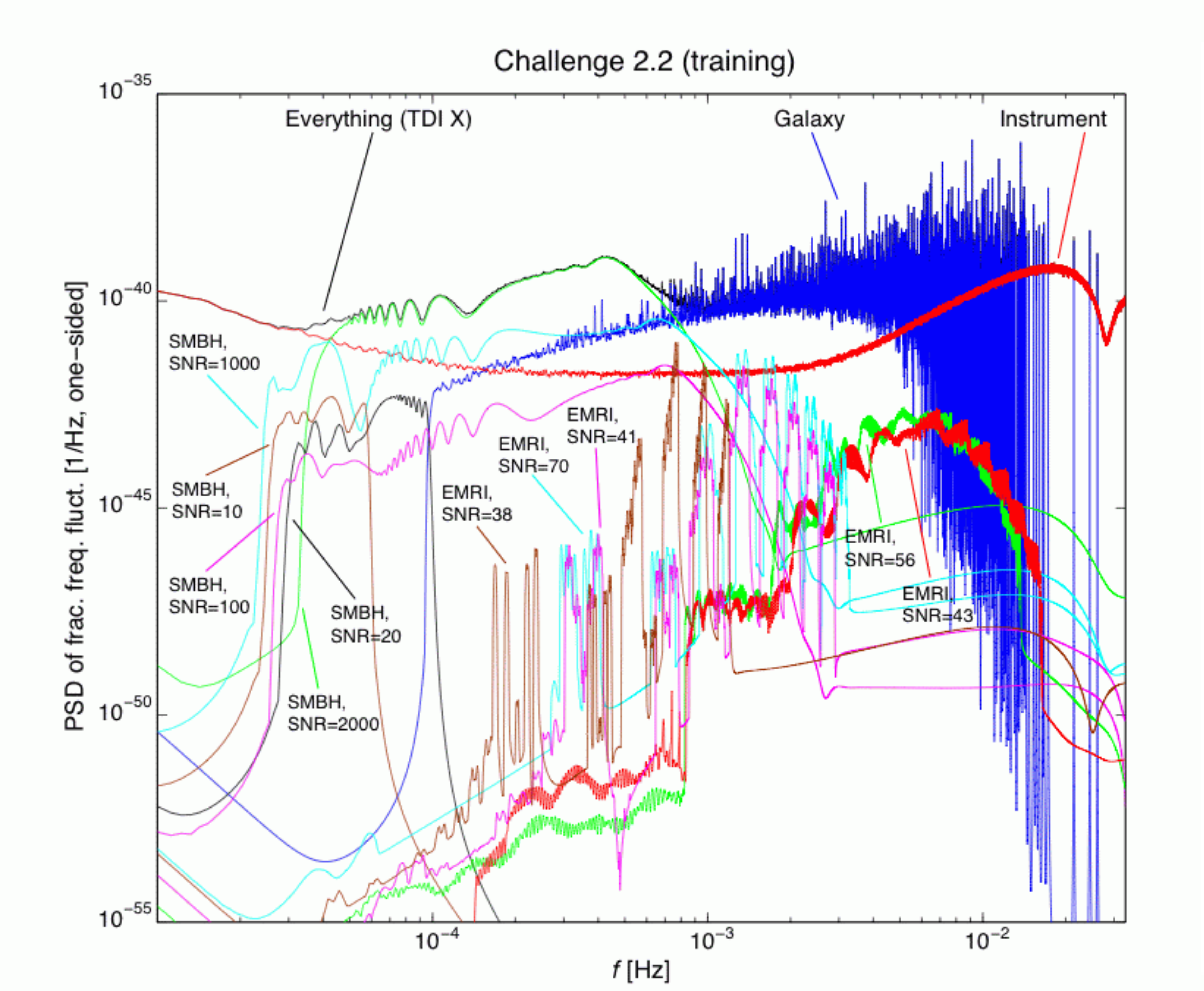
gravitation in técnico

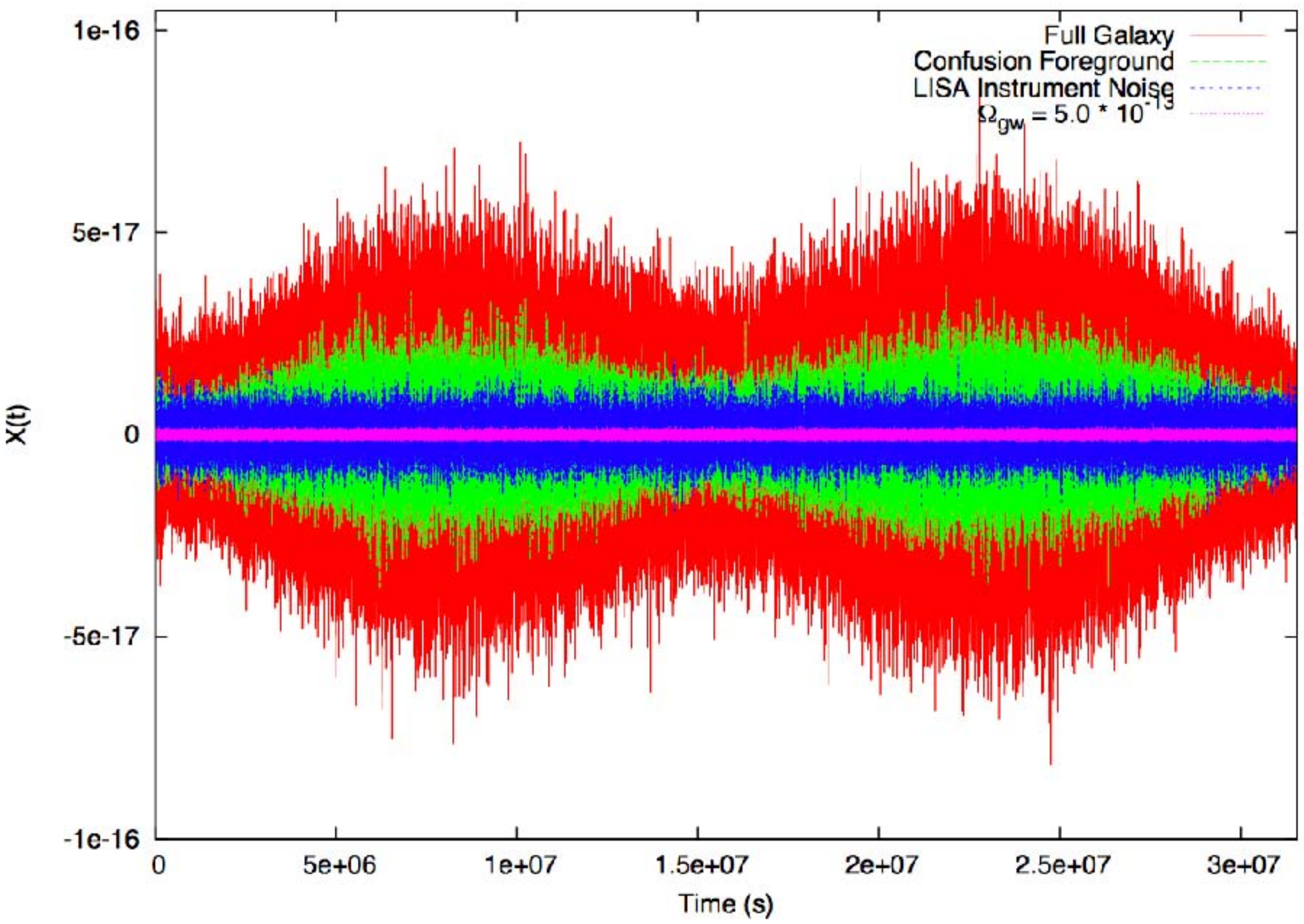
LISA is not LIGO in space



Thousand of overlapping signals

Many complex signals with multiple modulation timescales and harmonics

Current templates inadequate for accurate removal of bright signals



Noise is non-stationary on timescale of the signals

No off-source data for independent noise estimation

No second detector to veto noise transients

Signal Detection

data = signal + noise

$$d = h + n$$

(randomly selected aLIGO data from 14 September 2015)

Signal Detection



data - signal = noise

$$d - h = n$$

$$p(d|h) = p(d-h) = p(n)$$

The Global Solution

Likelihood function for Gaussian noise

$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det \mathbf{C}}}\, e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

$$\mathbf{d} = \text{data}$$

$$\mathbf{h} = \sum_{i=1}^{N} \mathbf{h}_i = \text{GW signal model}$$

$$\mathbf{C} = \text{noise correlation matrix}$$

$$\vec{\lambda} = \text{model parameters (signals, noise)}$$

$$M = \text{size of data}$$

The Global Solution

Likelihood function for Gaussian noise

$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det\mathbf{C}}}\, e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

$$\vec{\lambda} = \text{model parameters (signals, noise)}$$

$$\dim[\vec{\lambda}] \approx (140,000)_{\rm GB} + (1500)_{\rm SMBHB} + (1500)_{\rm EMRI} + (1000)_{\rm SOBH} + (2000)_{\rm N} \approx 146,000$$

Note: Number of resolvable sources *a priori* unknown - requires trans-dimensional model selection

The Global Solution

$$\chi^2 = (\mathbf{d} - \mathbf{h}) \cdot \mathbf{C}^{-1} \cdot (\mathbf{d} - \mathbf{h}) = (\mathbf{d} - \mathbf{h} | \mathbf{d} - \mathbf{h})$$

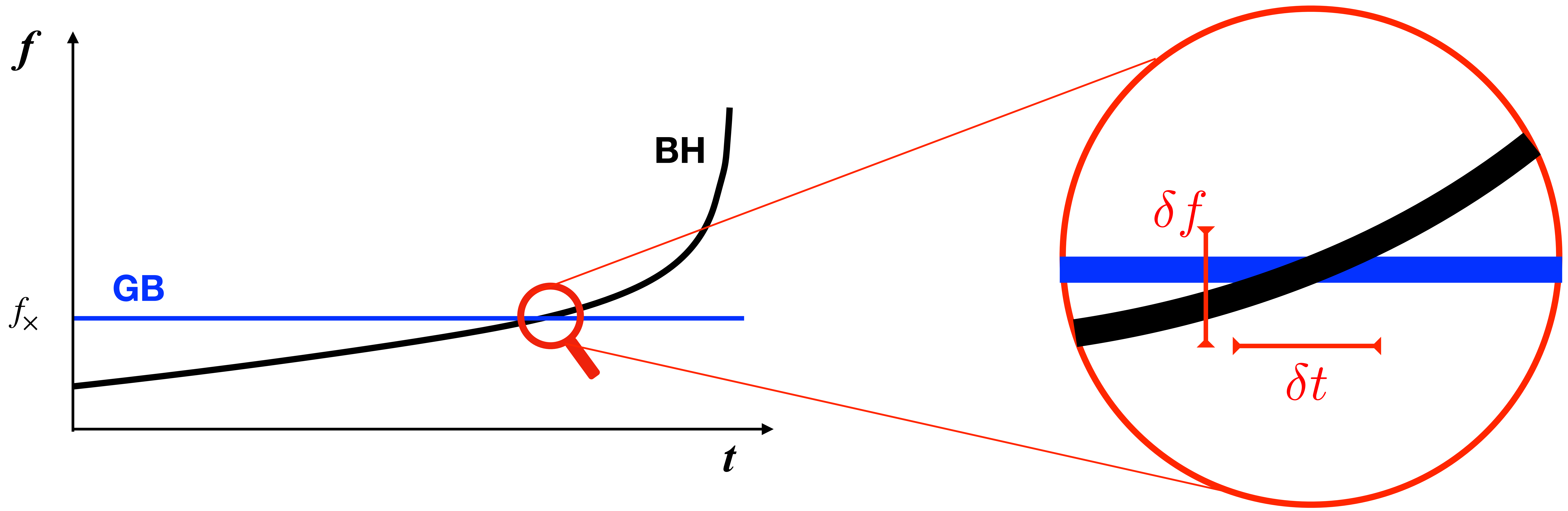
$$\chi^2 = \sum_i (\mathbf{d} - \mathbf{h}_i | \mathbf{d} - \mathbf{h}_i) + (N-1)(\mathbf{d}|\mathbf{d}) - \sum_{i \neq j} (\mathbf{h}_i | \mathbf{h}_j)$$

Individual source chi-squared

Overlap between sources

If the overlap terms were zero we wouldn't need a global solution

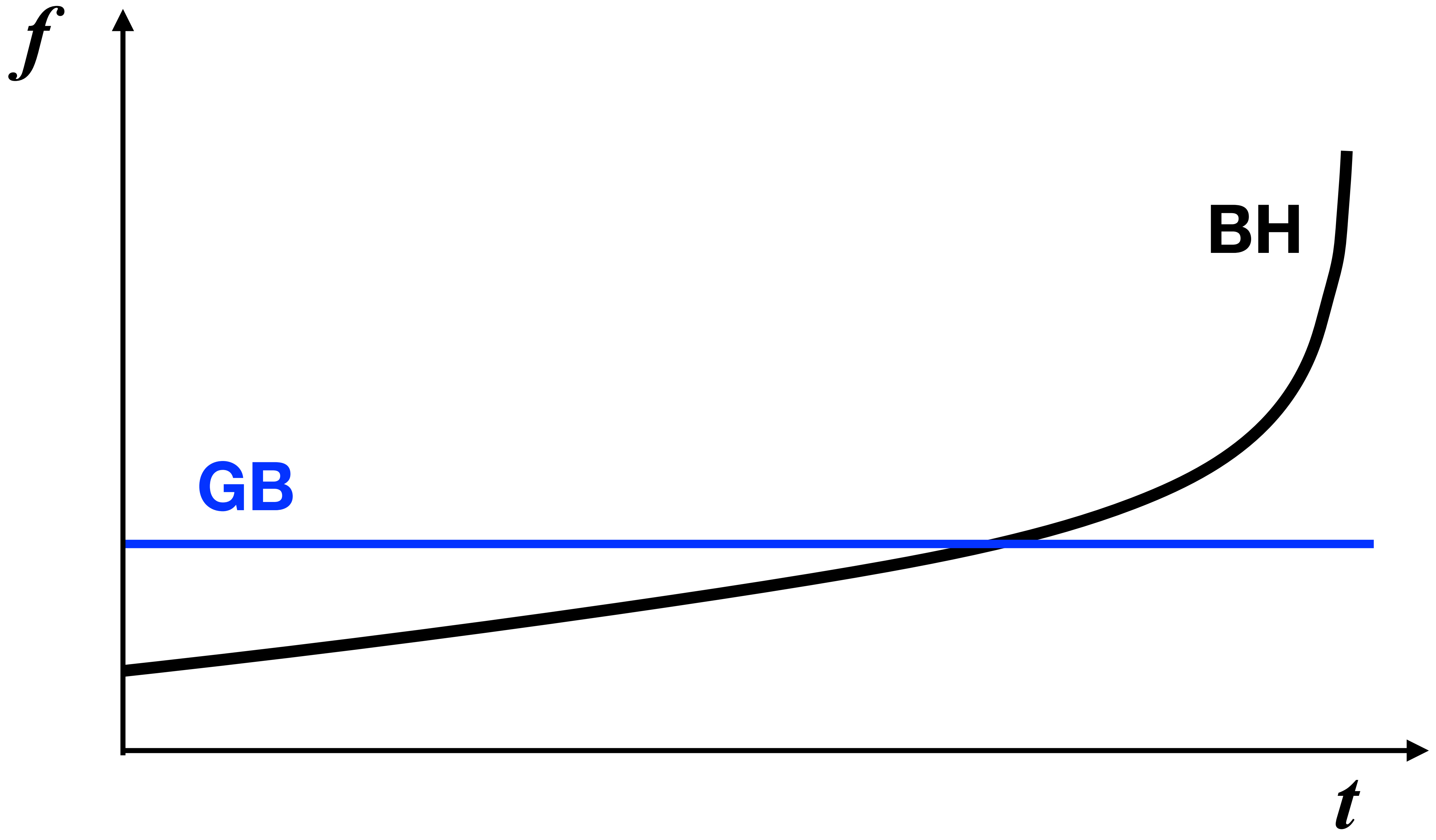
Signal Overlaps, SMBH-GB



$$\delta f = 3\times 10^{-7}\,\mathrm{mHz}\left(\frac{\mathcal{M}_{\mathrm{GB}}}{0.25 M_\odot}\right)^{5/6}\left(\frac{f_\times}{1\,\mathrm{mHz}}\right)^{11/6}$$

$$\delta t = 1.7\times 10^{3}\,\mathrm{s}\left(\frac{10^6 M_\odot}{\mathcal{M}_{\mathrm{BH}}}\right)^{5/6}\left(\frac{1\,\mathrm{mHz}}{f_\times}\right)^{11/6}$$

Signal Overlaps, SMBH-GB



Instantaneous BH SNR

Random phase mismatch

$$(\mathbf{h}_{\rm BH}|\mathbf{h}_{\rm GB}) = \left(\frac{S_{\rm BH}(f_\times)}{S_{\rm n}(f_\times)}\right)^{1/2} \rho_{\rm GB}\, \cos\delta$$

$$\approx 10^{-3}\, \rho_{\rm BH}\, \rho_{\rm GB}\, \cos\delta$$

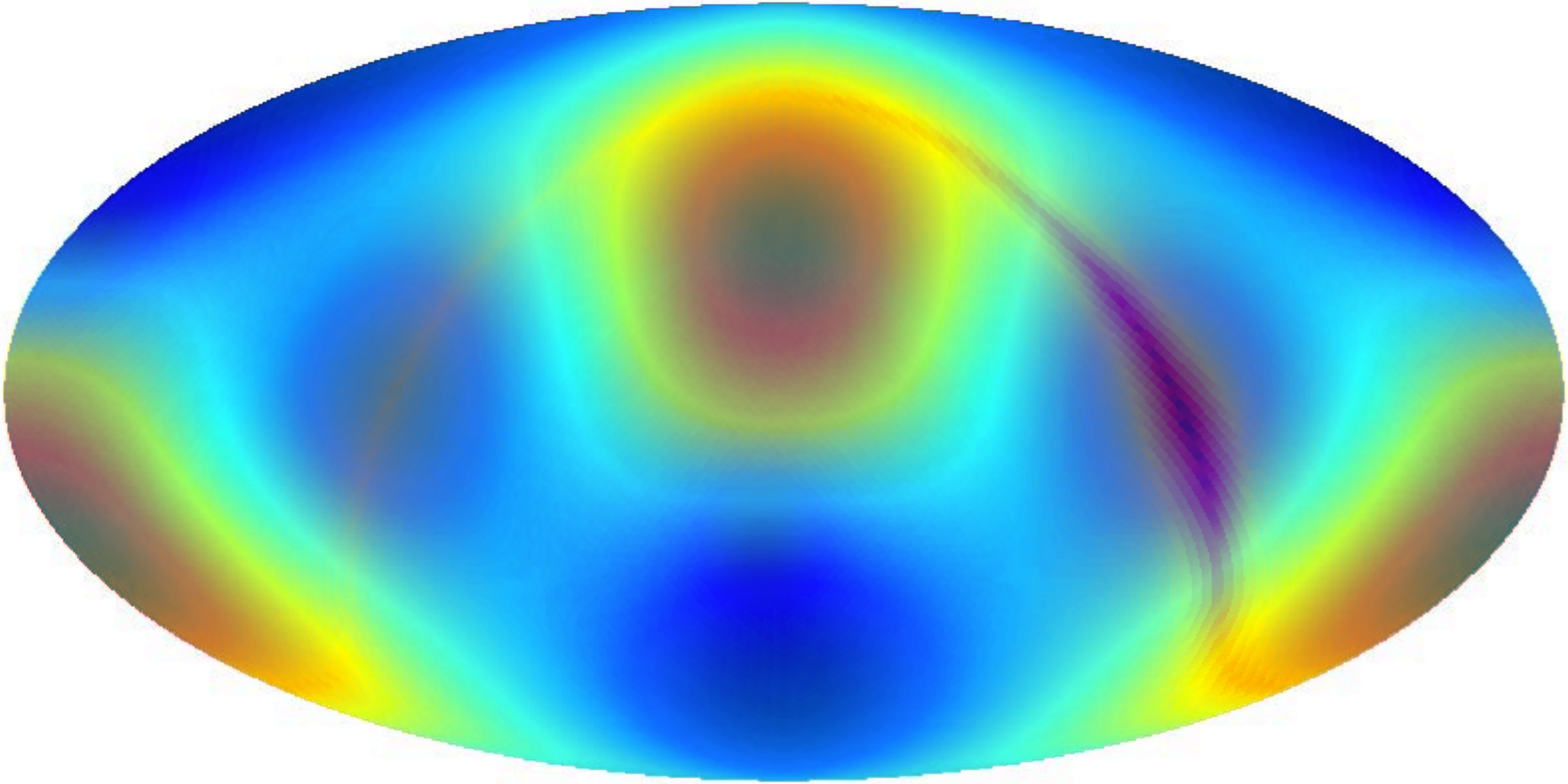
Individual overlaps are small, but there will be tens of thousands of them

$$\sum_{\rm GB} (\mathbf{h}_{\rm BH}|\mathbf{h}_{\rm GB}) \approx \sqrt{N}_{\rm GB}\, 10^{-3}\, \rho_{\rm BH}\, \rho_{\rm GB} \approx 0.2\, \rho_{\rm BH}\, \rho_{\rm GB}$$

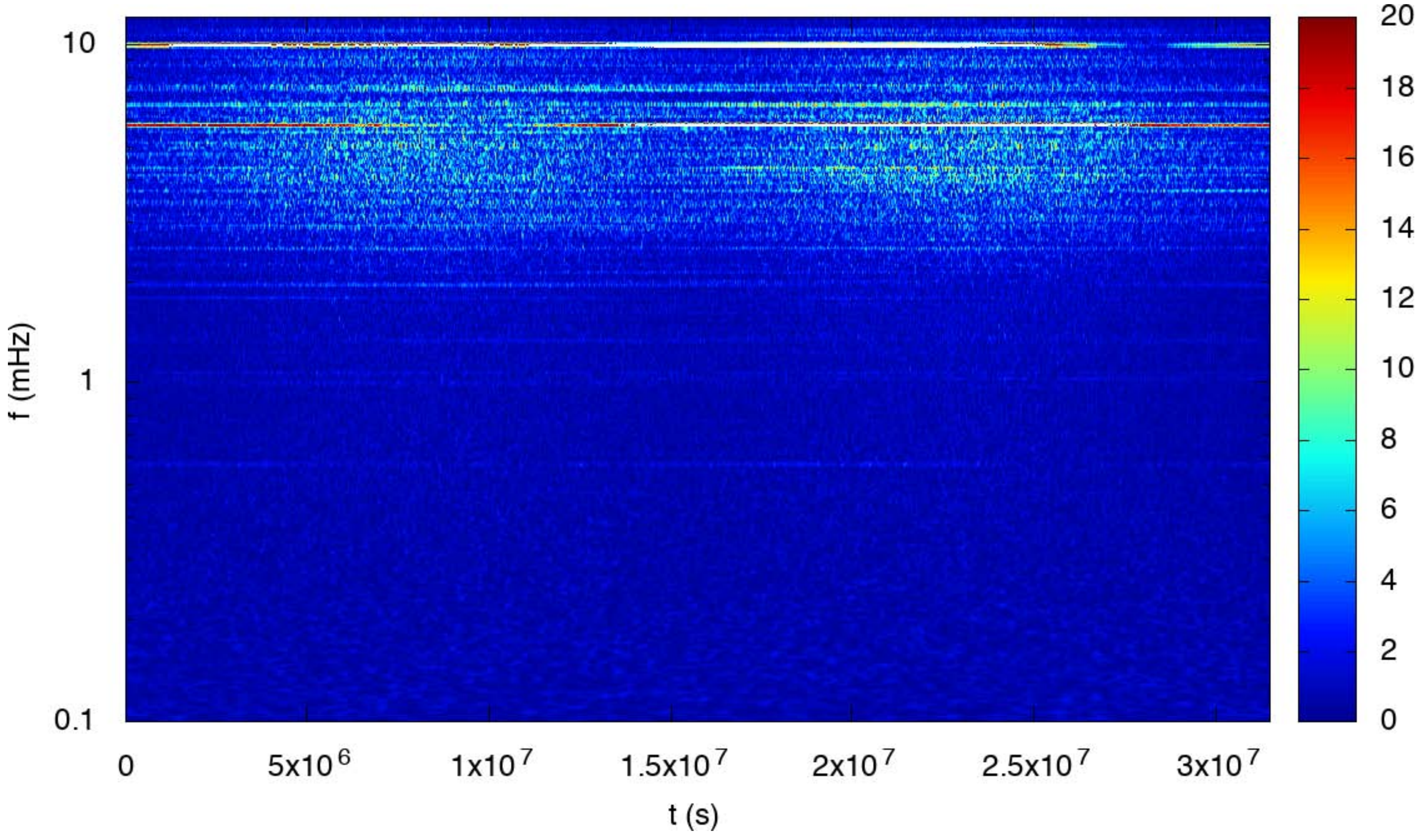
Significant bias if not solved for simultaneously

[see Cutler & Harms (2008), Robson & Cornish (2018)]

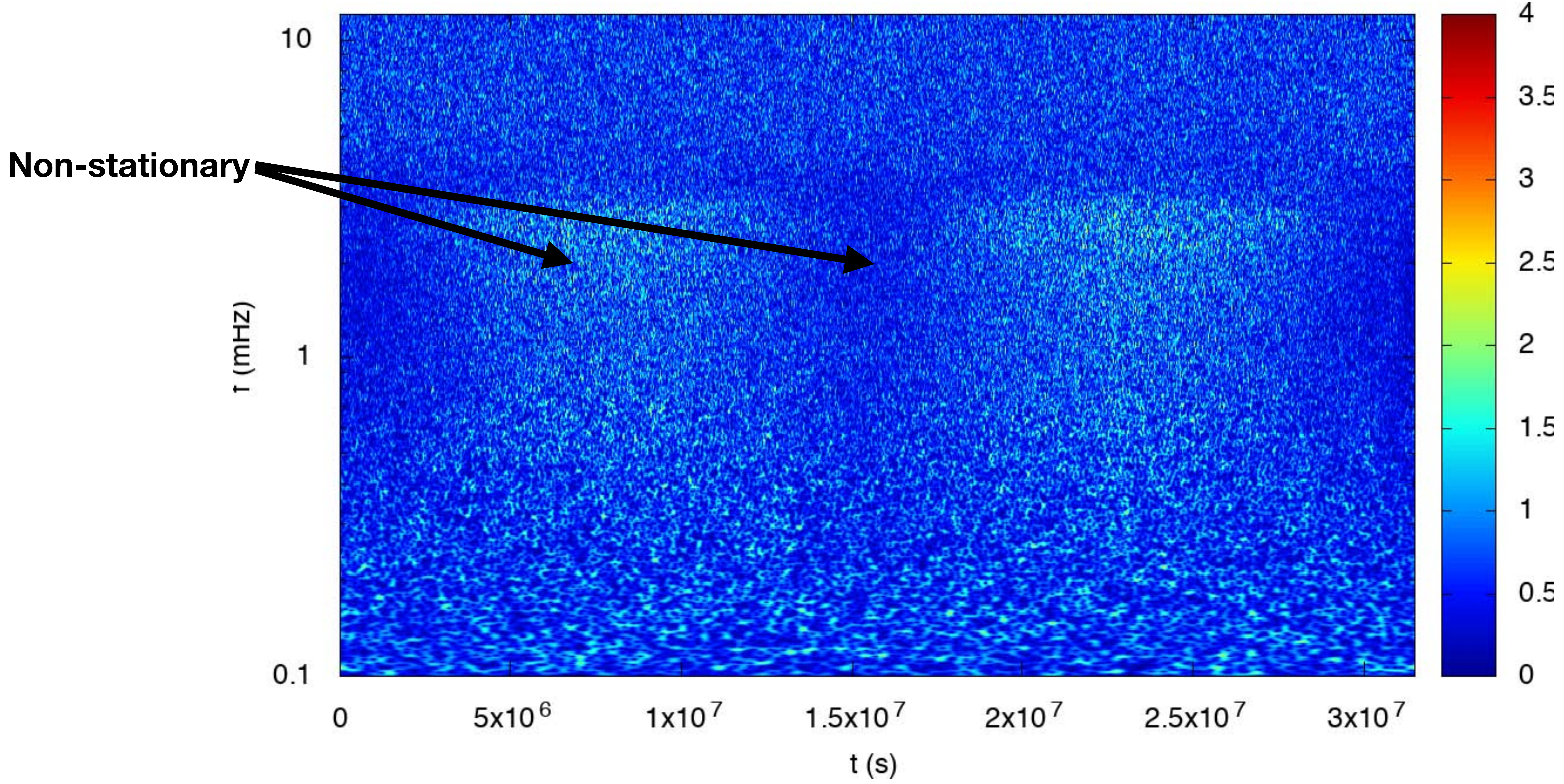
Non-Stationary Data



Whitened Galaxy + Instrument noise spectrogram, 1 year



Whitened Galactic Confusion + Instrument noise spectrogram, 1 year

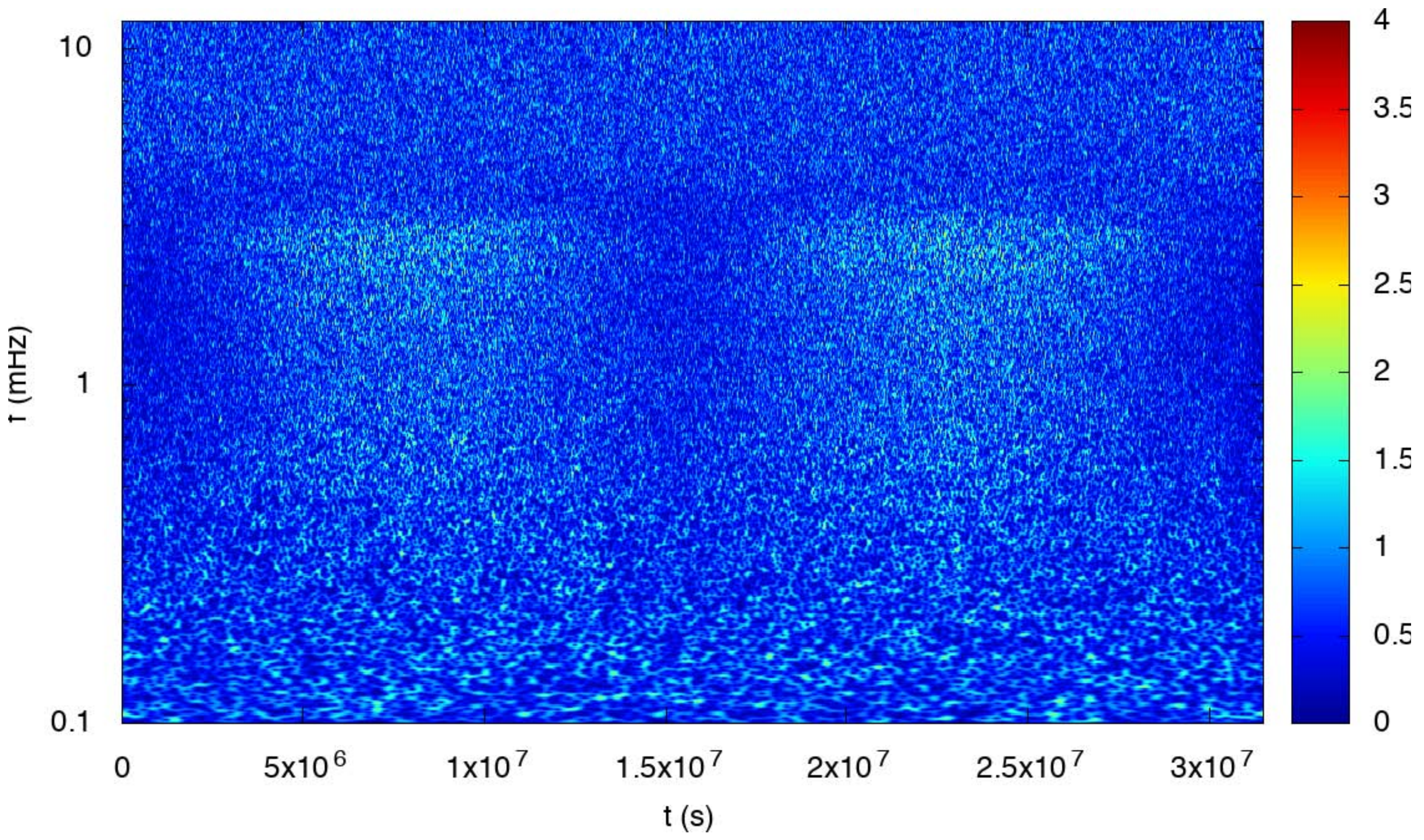


Likelihood for Non-stationary Gaussian Noise

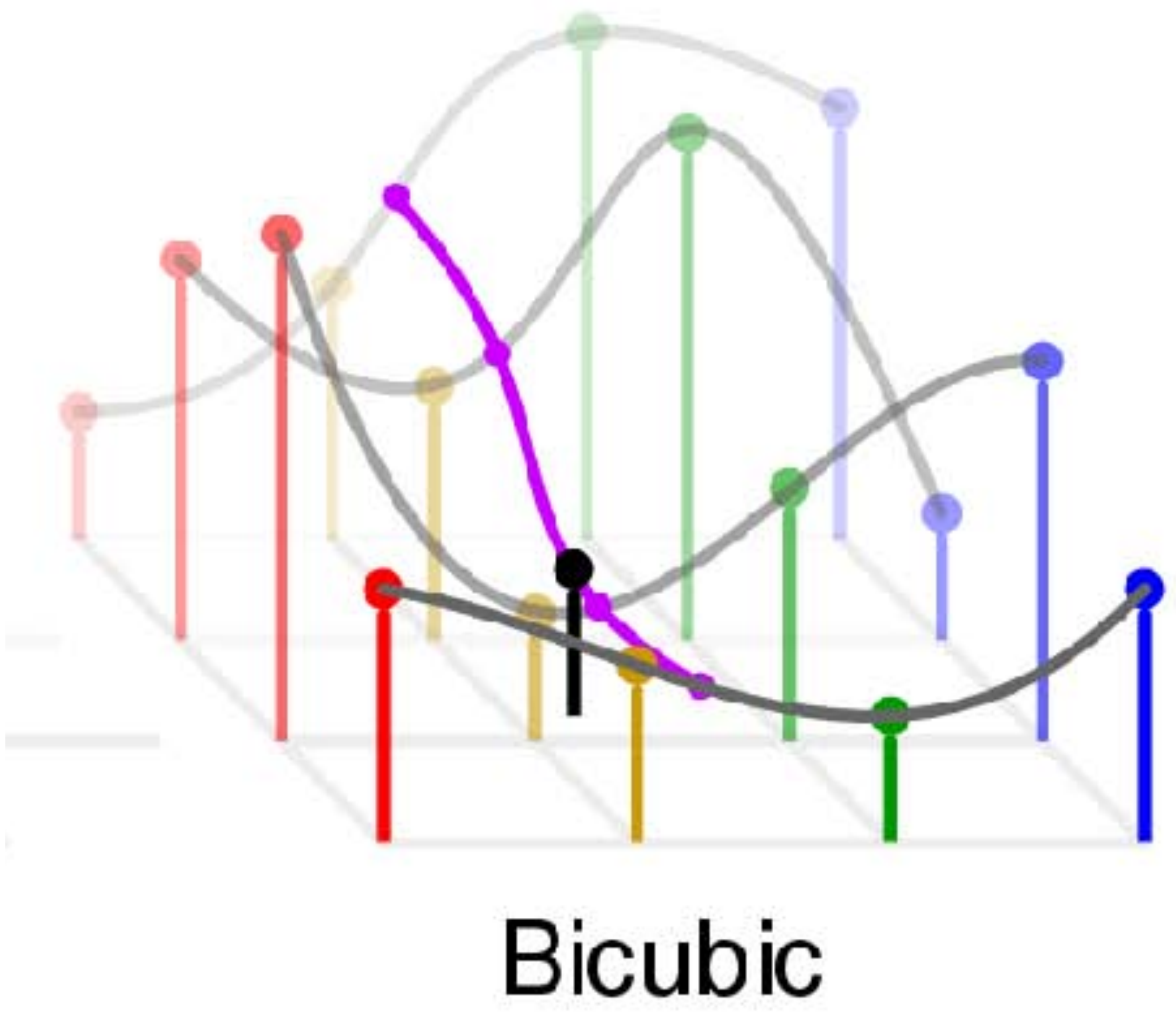
$$p(\mathbf{d}|\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^\dagger \mathbf{C}^{-1}(\mathbf{d}-\mathbf{h})}$$

Use a discrete wavelet based likelihood for LISA data analysis

$$C_{(i,j)(k,l)} = \delta_{ij}\delta_{kl}\, C_{ik}$$

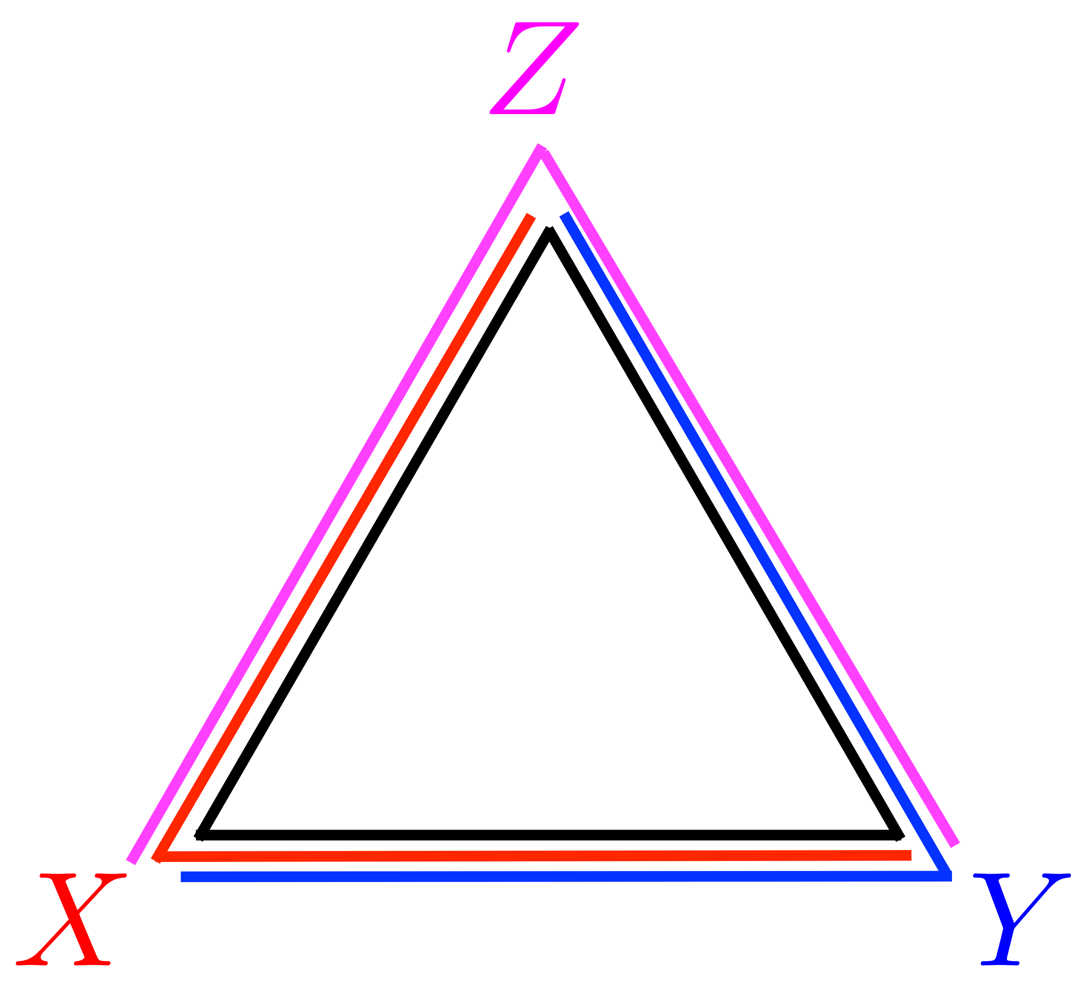


Model the wavelet spectrum C_{ik} as a smooth function in frequency and time. E.g. Trans-dimensional Bicubic spline

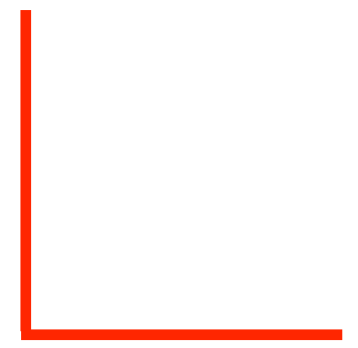


[Cornish, in prep. (2018)]

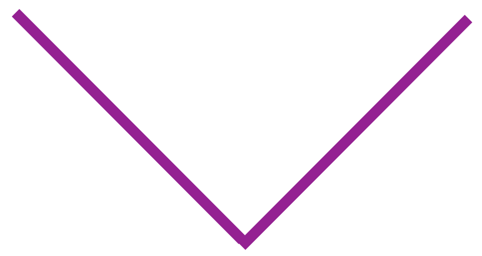
Detecting Un-modeled Signals



$$S_+ = \frac{\sqrt{3}}{2}\, X \quad \Rightarrow$$



$$S_\times = \frac{1}{2}\,(X + 2Y) \quad \Rightarrow$$

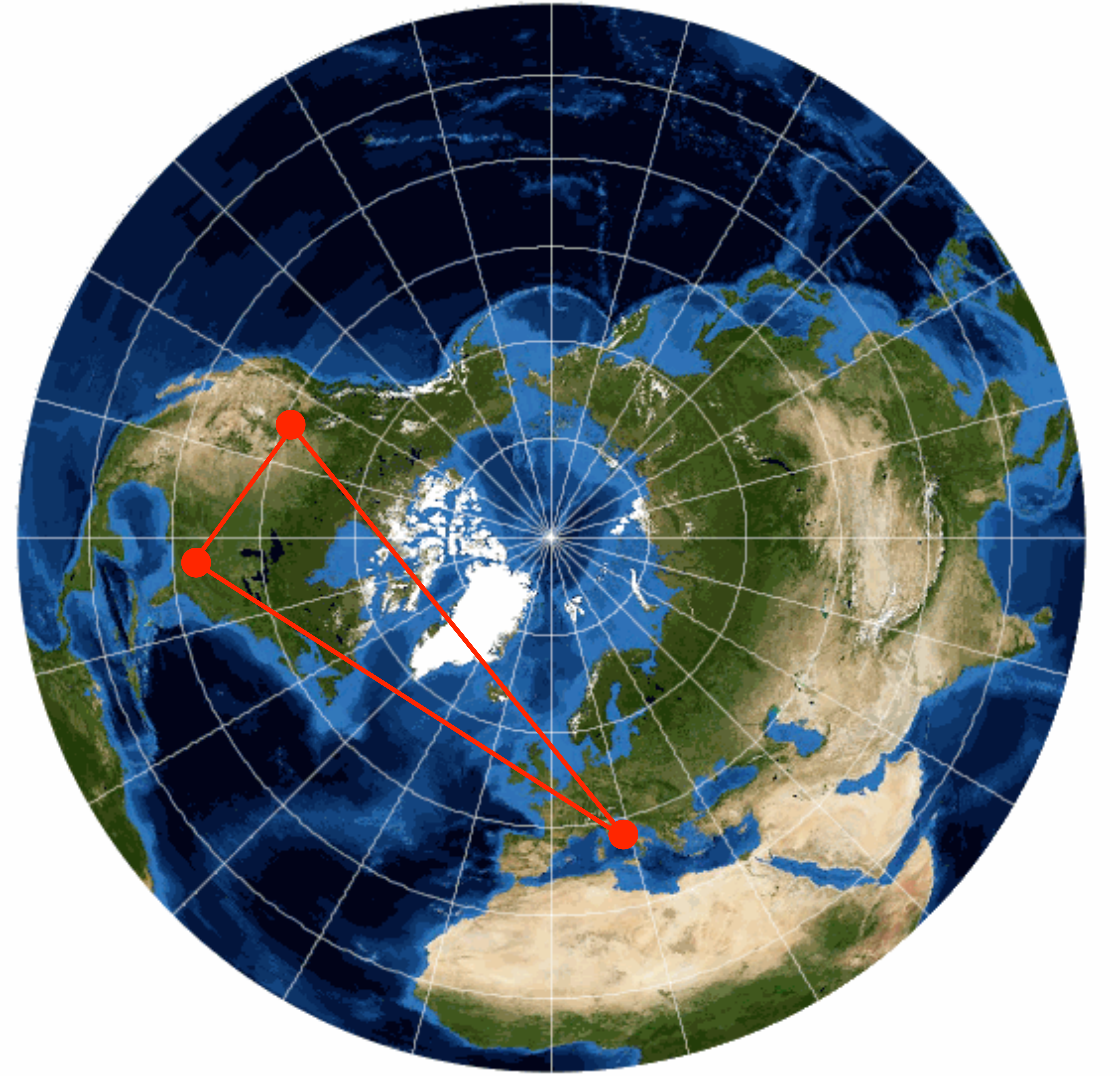


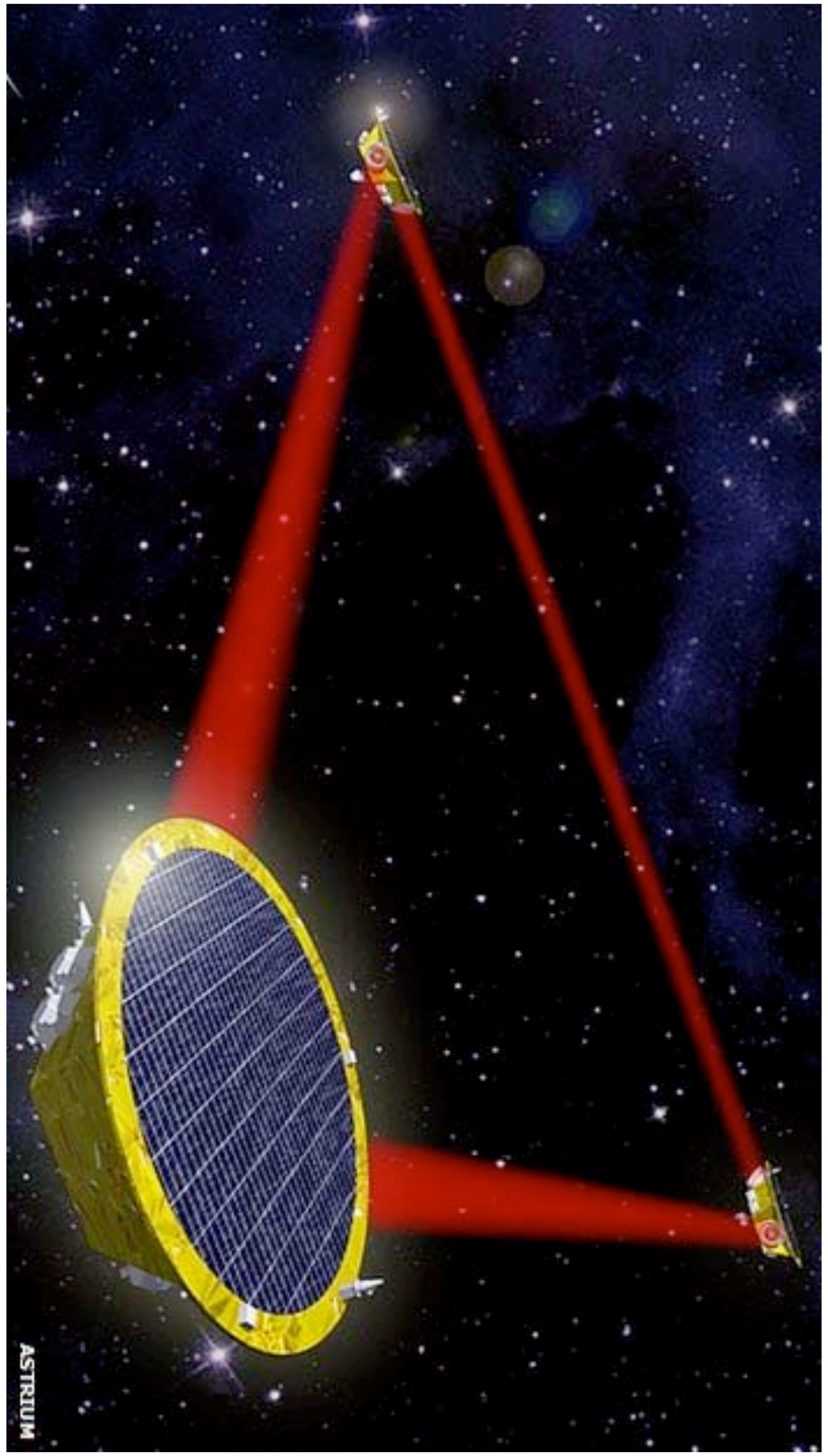
Sensitive to GWs and glitches

$$S_\odot = \frac{1}{3}\,(X + Y + Z) \quad \Rightarrow$$

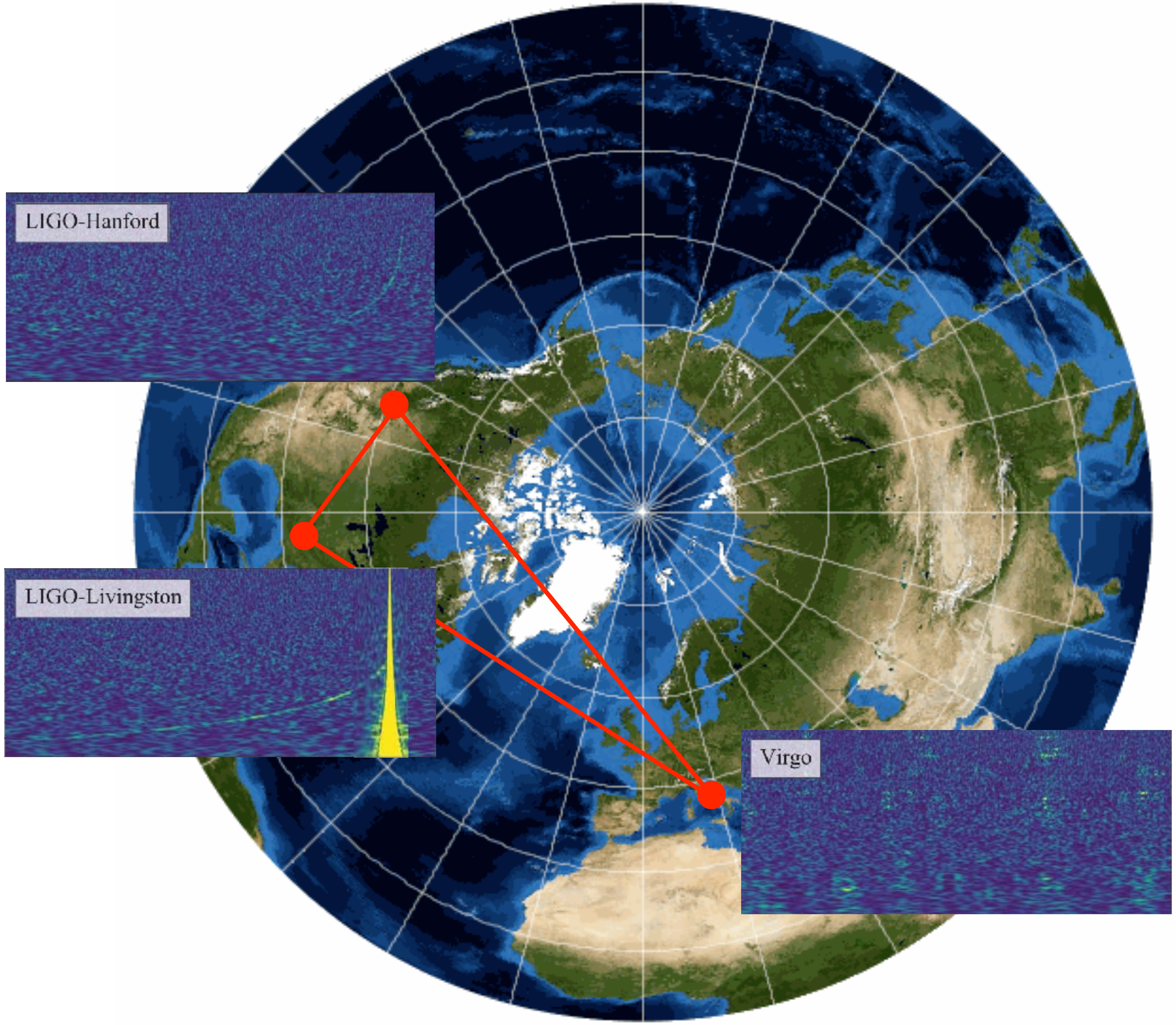
Insensitive to GWs, Sensitive to Glitches

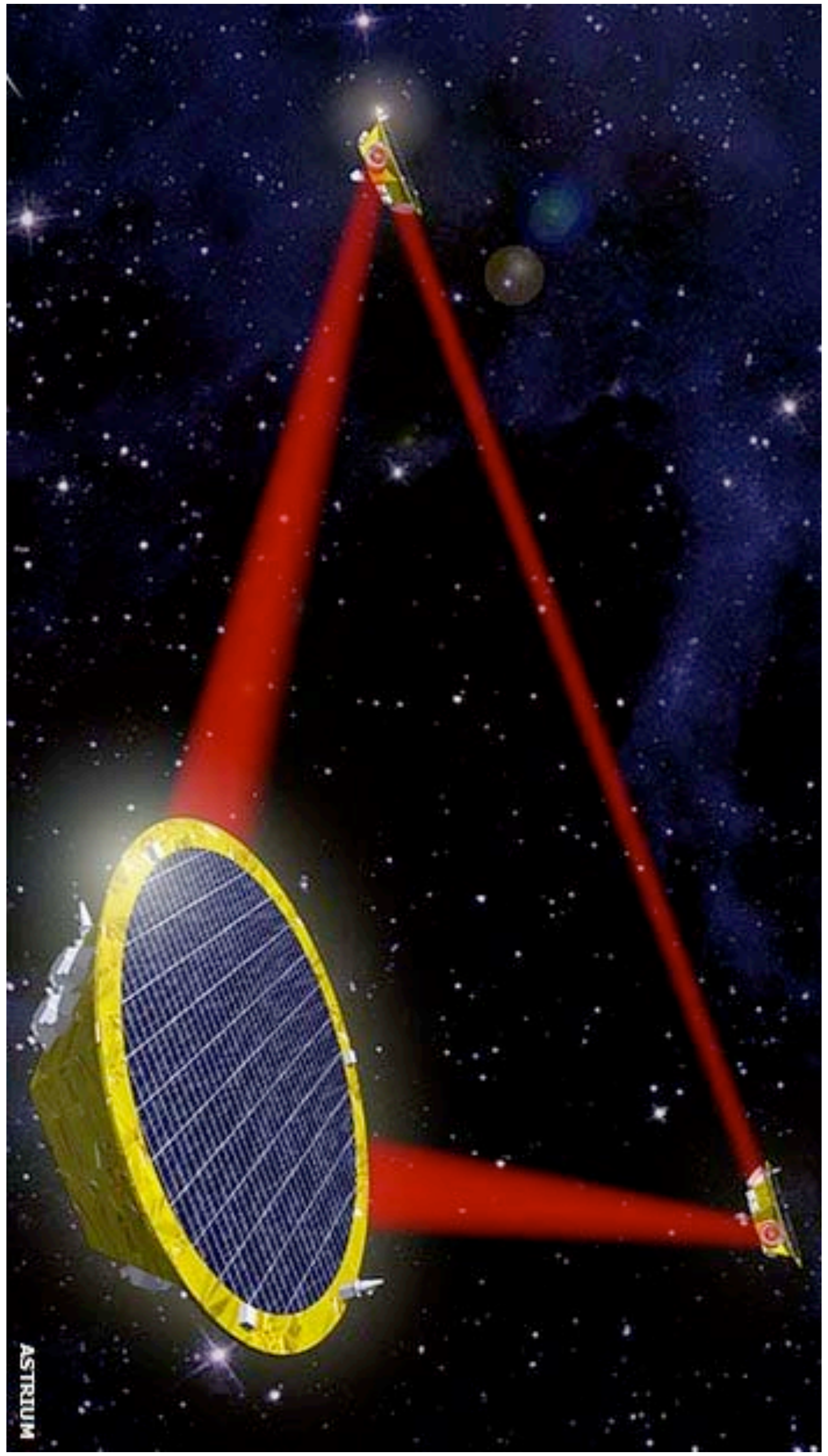
Detecting Un-modelled signals



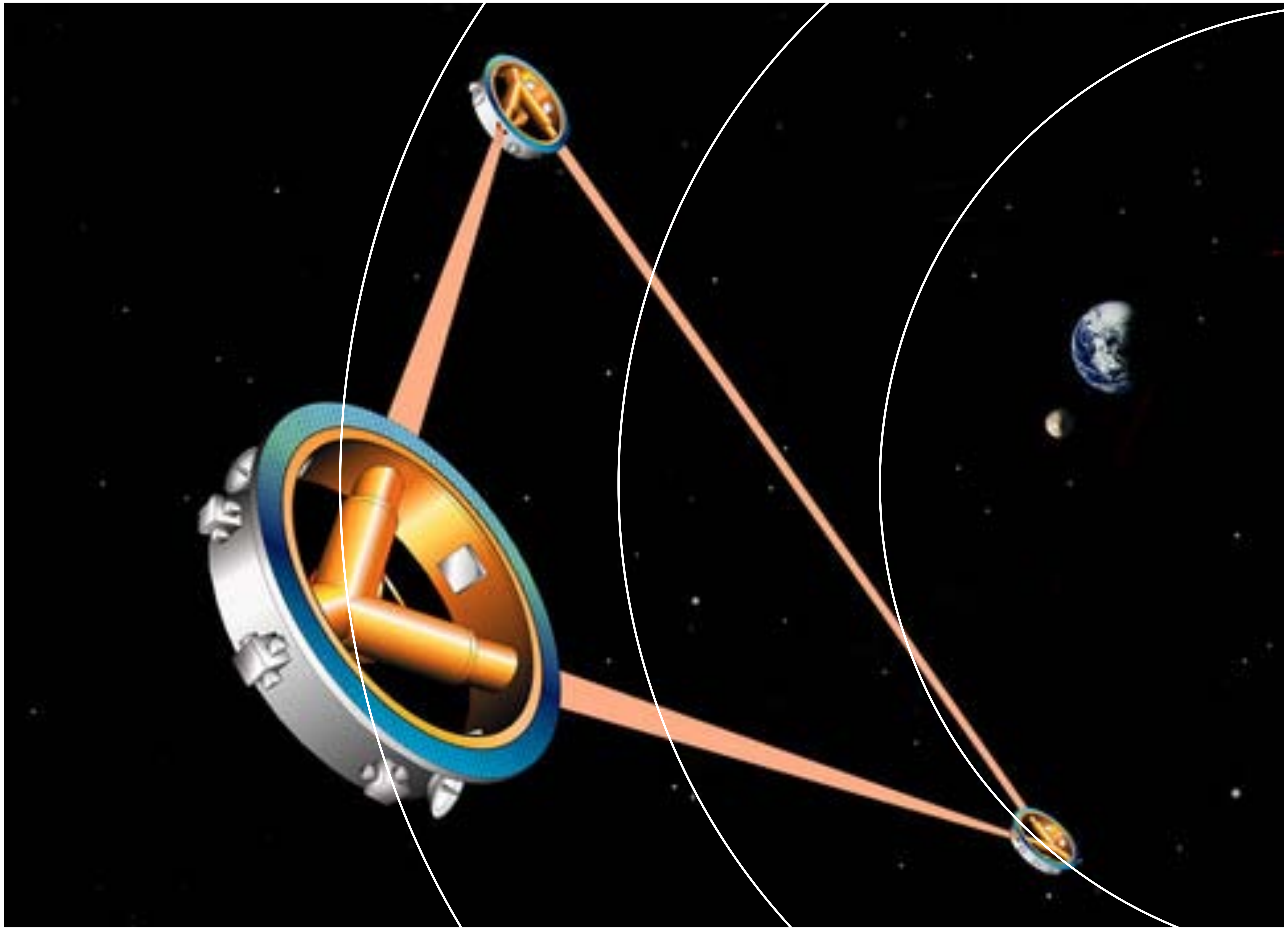


Detecting Un-modelled signals





Separating Burst Signals from Noise



Noise delays

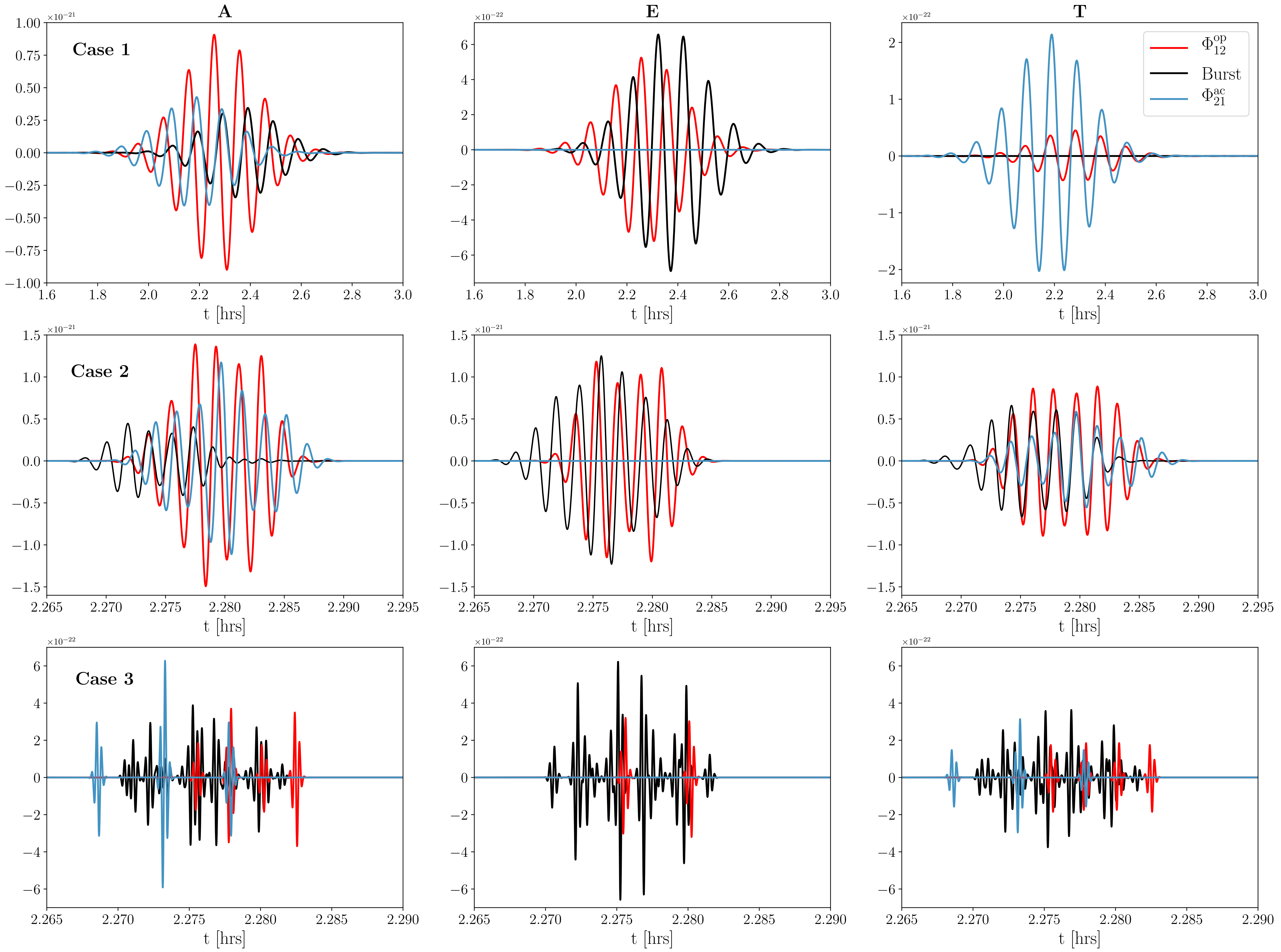
$$\Delta t = n\,\frac{L}{c}$$

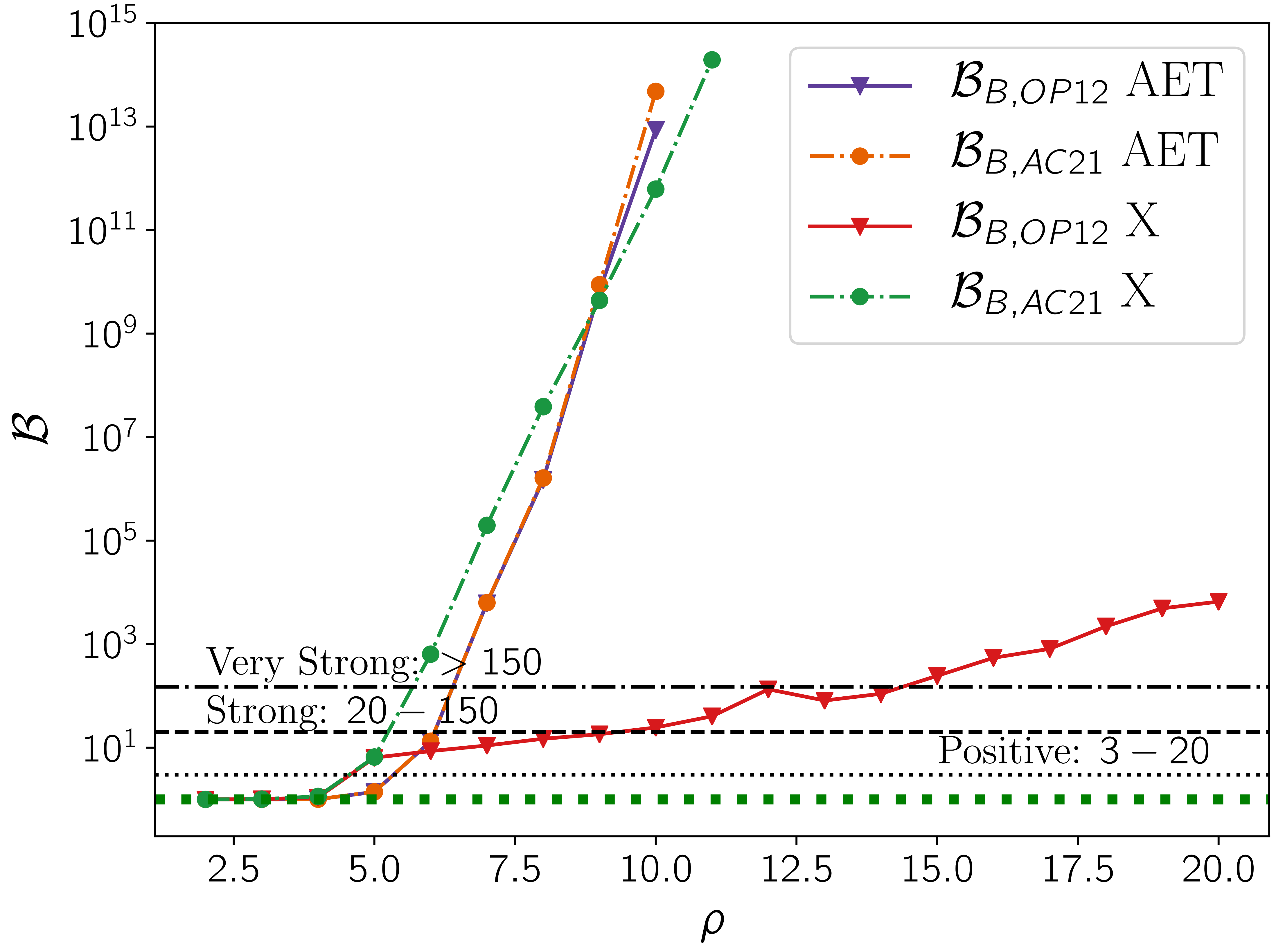
Signal delays

$$\Delta t = n\,\frac{L}{c} + \frac{\hat{k}\cdot\vec{L}}{c}$$

[Robson & Cornish (2018)]

Separating Burst Signals from Noise

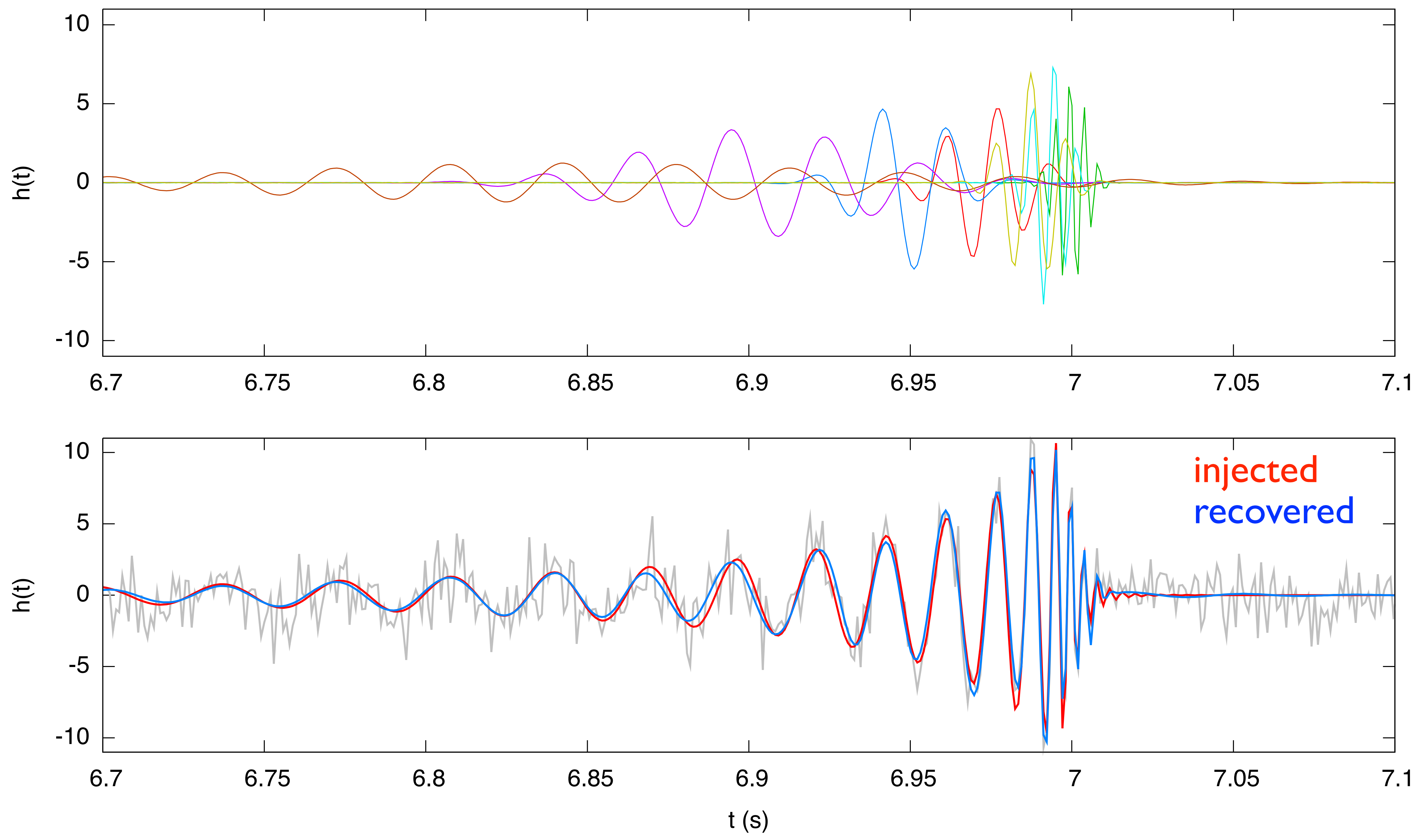




[Robson & Cornish arXiv:1811.04490]

Detecting signals with arbitrary morphology

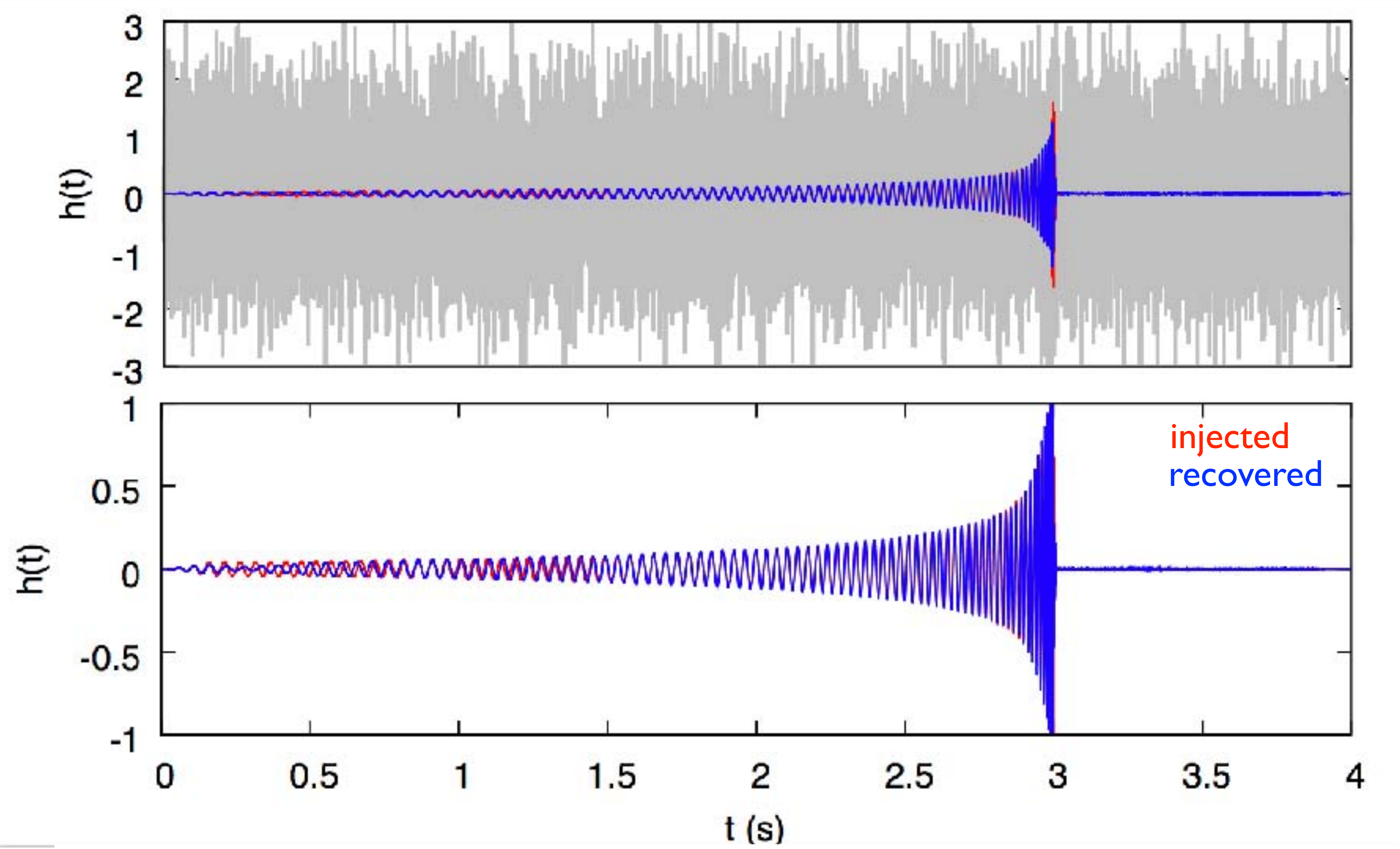
BayesWave (Wavelets)



SNR = 28, Match = 0.95, Mtotal = 75 Msun

$$h(t) = \sum_{k=0}^{N_w} \psi(t, \boldsymbol{\theta}_k)$$

BayesWave (Splines)



SNR = 15, Match = 0.95, Mtotal = 35 Msun

$$h(t) = \sum_{k=0}^{N_h} \sum_{j=2}^{N_a} A_k(t, \boldsymbol{\theta}_j) \cos\left(\int 2\pi \sum_{l=2}^{N_f} f_k(t, \boldsymbol{\lambda}_l) dt \right)$$

Stochastic Signals - Subtraction?

Likelihood

$$p(\mathbf{d}|\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^M \det\mathbf{C}}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})\cdot\mathbf{C}^{-1}\cdot(\mathbf{d}-\mathbf{h})}$$

Prior

$$p(\mathbf{h}) = \frac{1}{\sqrt{\det(2\pi\mathbf{S_h})}} e^{-\frac{1}{2}(\mathbf{h}^\dagger\mathbf{S_h}^{-1}\mathbf{h})}, \quad p(\mathbf{S_h})$$

We are not interested in the individual random values of **h,** so marginalize over **h** [Cornish & Romano (2013)]

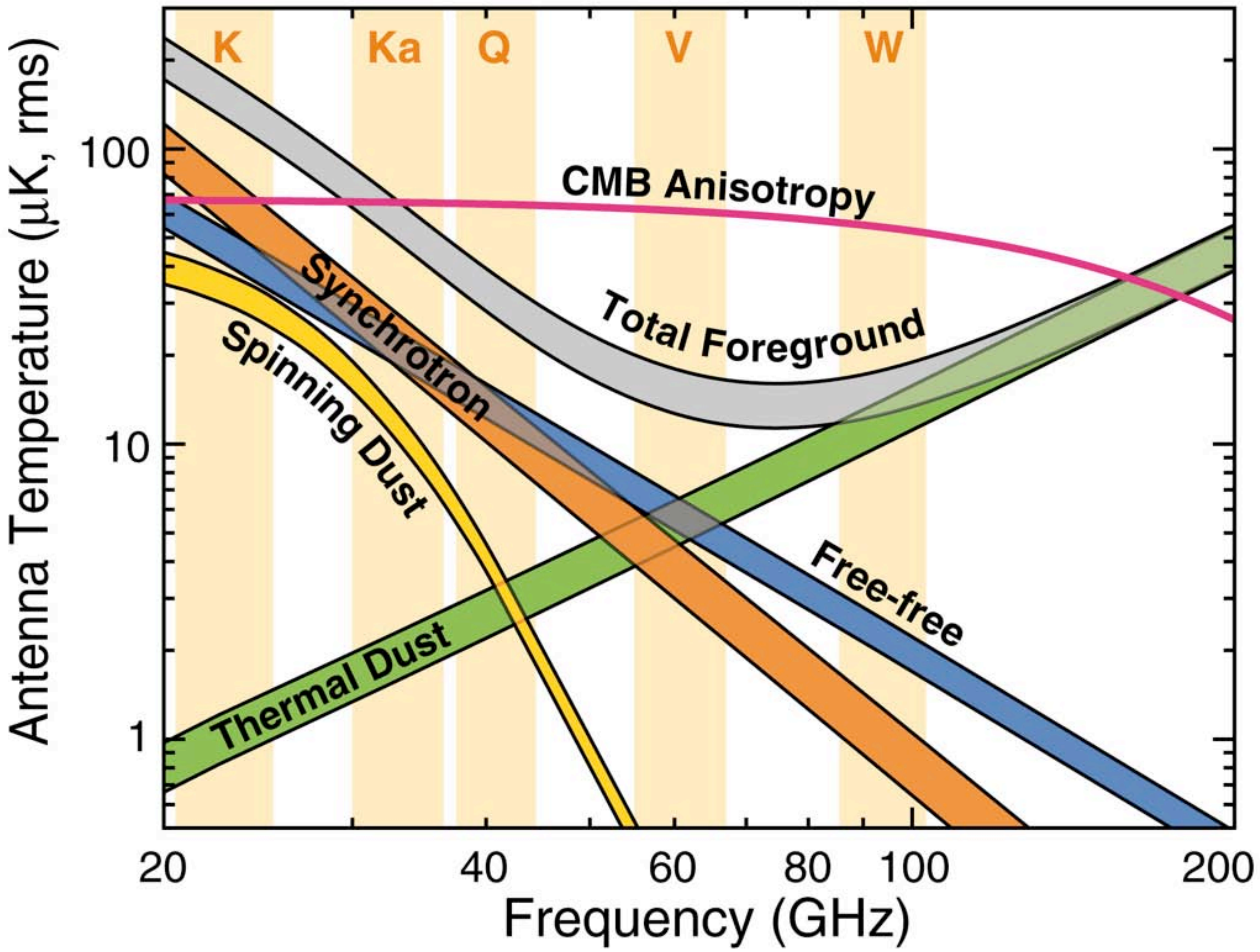
$$\mathbf{C} \to \mathbf{C} + \mathbf{H} \qquad H_{(Ia)(Jb)} = H_{IJ}\, S^h_{ab}$$

Overlap reduction function

GW power spectrum

It is subtraction, but the subtraction is not deterministic - better thought of as spectral modeling

CMB Spectral Component Separation



Spectral separation for LISA

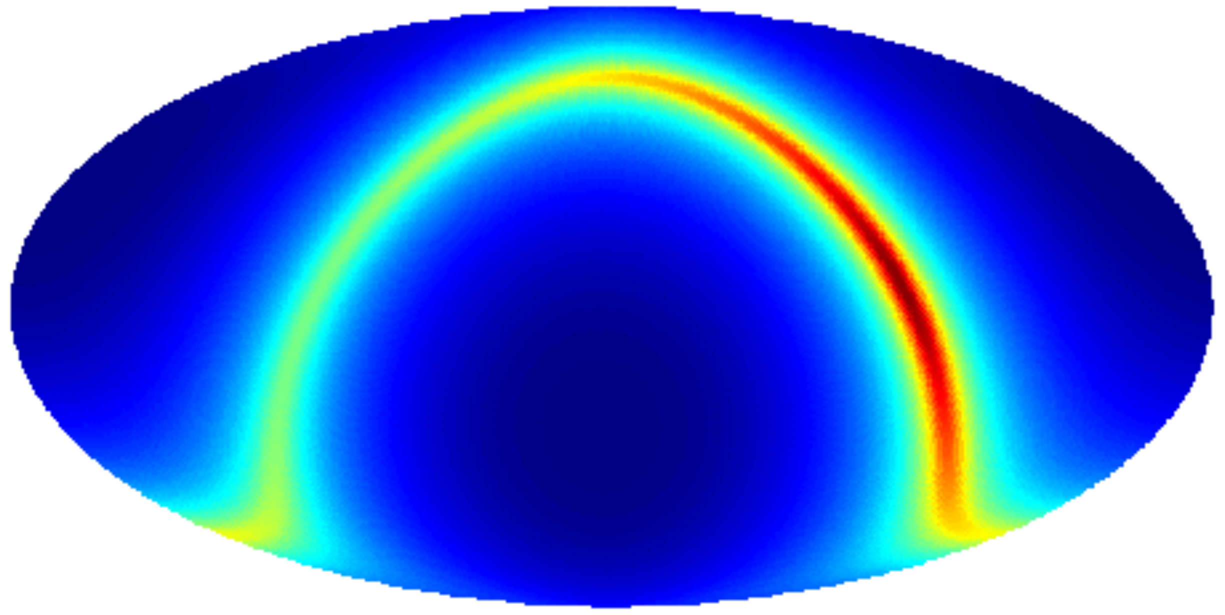
Likelihood $$p(X|S_a, S_p, S_h) = \prod_f \frac{1}{(2\pi)^{3/2}|C|} e^{-(X_i C_{ij}^{-1} X_j)/2}$$

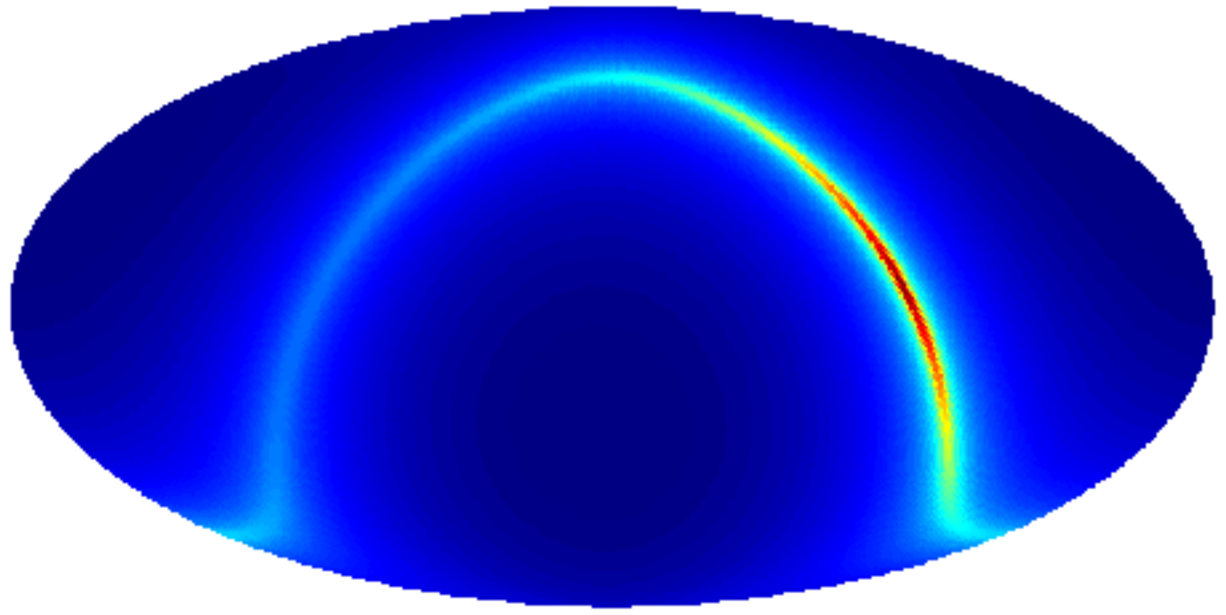
$$\text{where} \quad C(f) = \begin{pmatrix} \langle AA \rangle & \langle AE \rangle & \langle AT \rangle \\ \langle EA \rangle & \langle EE \rangle & \langle ET \rangle \\ \langle TA \rangle & \langle TE \rangle & \langle TT \rangle \end{pmatrix}$$

Dominated by noise, insensitive to signal

Spectral Priors $p(S_a),\ p(S_p)\ p(S_h)$

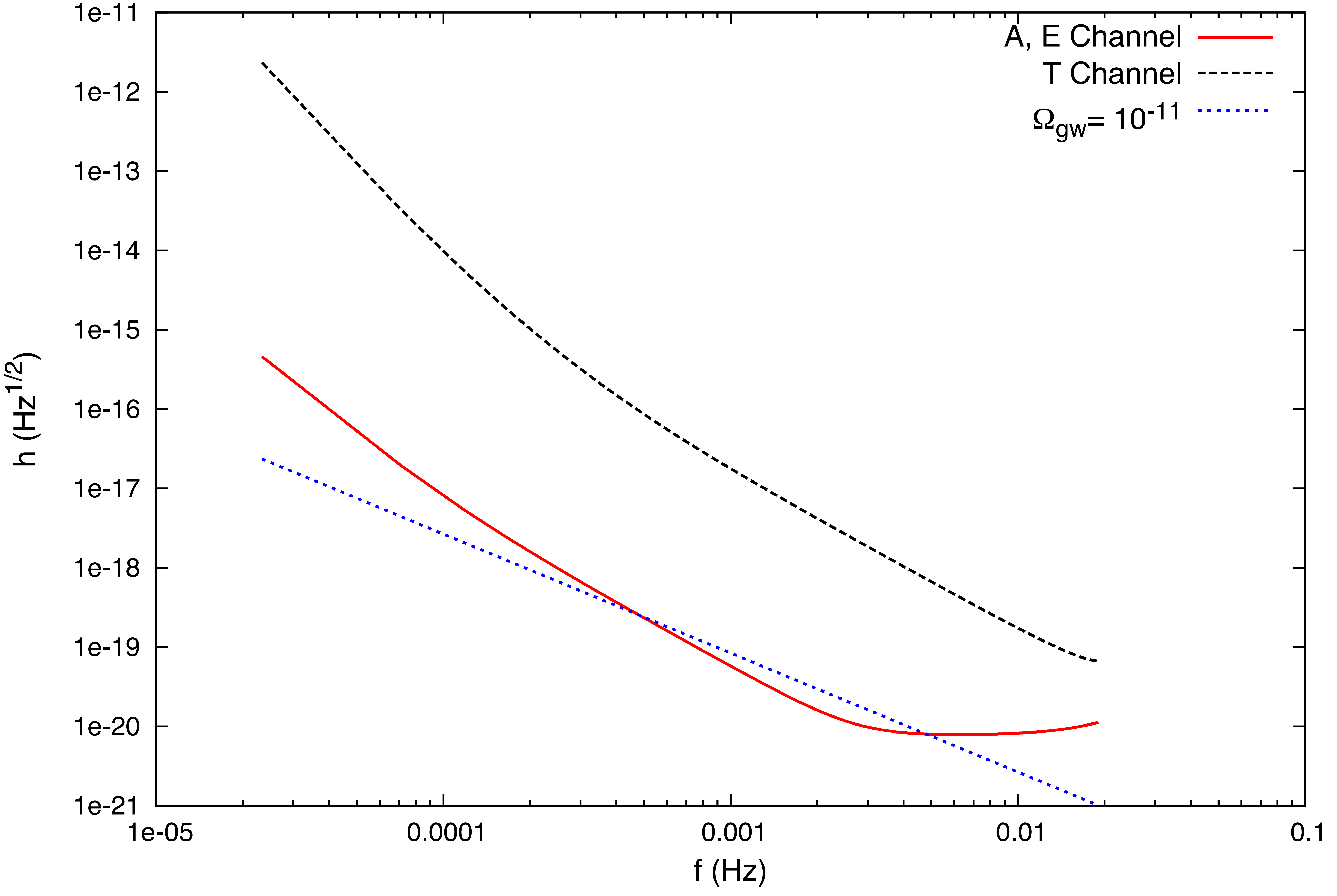
Galaxy Shape Priors $$\rho(x,y,z) = \rho_0\, e^{-\sqrt{x^2+y^2}/R_d}\, \mathrm{sech}^2(z/Z_d)$$

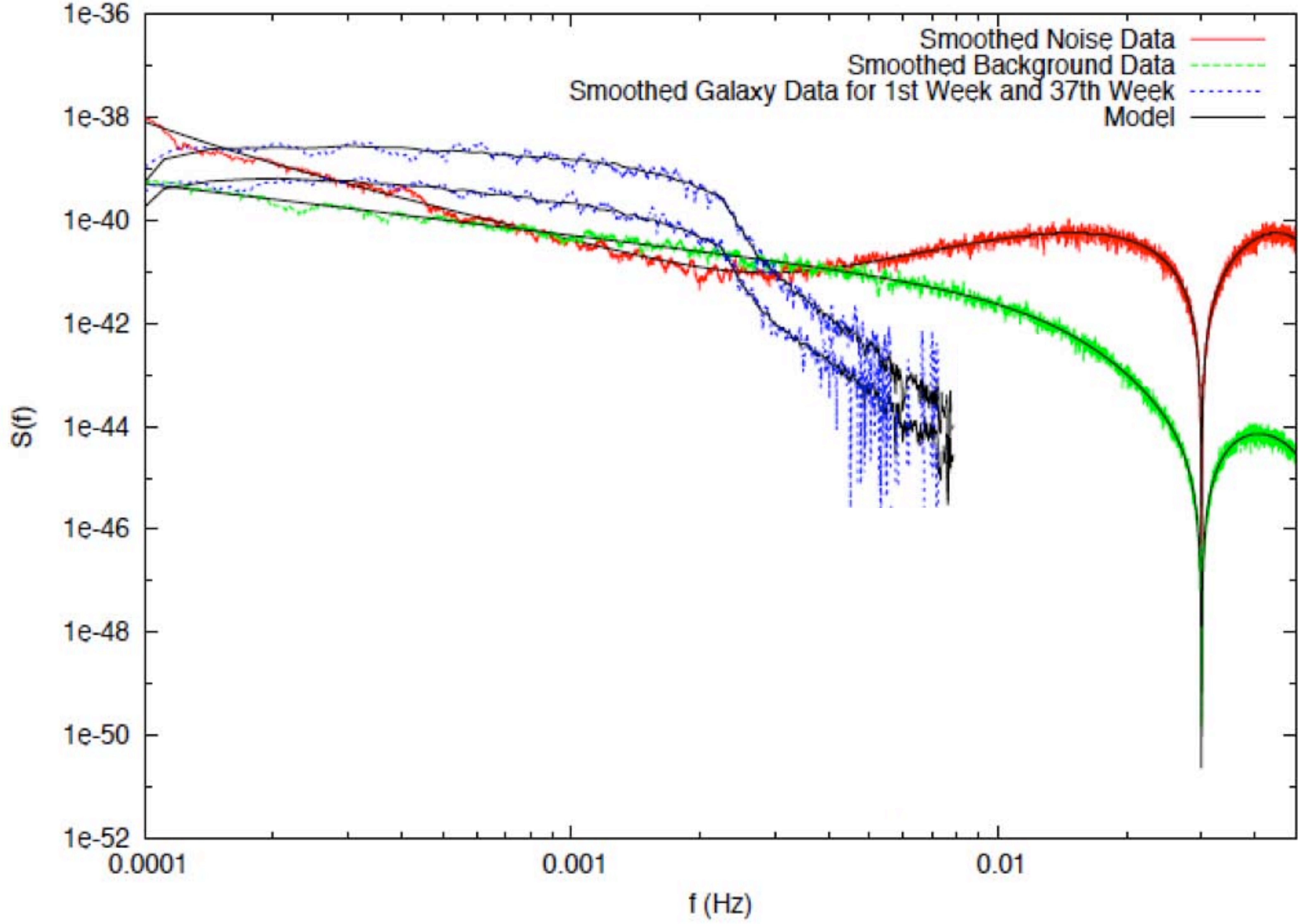




[Adams & Cornish (2014)]

LISA Sensitivity Curves





Smoothed Noise Data
Smoothed Background Data
Smoothed Galaxy Data for 1st Week and 37th Week
Model
1e-36
1e-38
1e-40
1e-42
1e-44
1e-46
1e-48
1e-50
1e-52
S(f)
0.0001
0.001
0.01
f (Hz)

Starting Points For Discussion

- The potential for bias from improper modeling of the instrument

- How to detect un-modeled signals?

- How to look for exotic signals or departures from GR while simultaneously modeling $\gtrsim 10^4$ sources?

- Model-Independent or Model-Specific tests of GR?

- New physics or environmental effects?