Theoretical Constraints and Effective Field Theory

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Overview

- Brief discussion of theoretical considerations about models, particularly relevant to deciding which ones to devote large amounts of computer/simulation time to …

  "Beyond the Cosmological Standard Model", B. Jain, A. Joyce, J. Khoury and MT, Phys. Rept. 568 1-98 (2015), [arXiv:1407.0059]

- Here, due to time, restrict to a single point. Discussion of when higher derivative terms are necessary in the effective field theory, and how, if not, they can be removed - useful again e.g. for simulating.

  "Higher-derivative operators and effective field theory for general scalar-tensor theories," A.R. Solomon and M.T., JCAP {\bf 1802}, no. 02, 031 (2018) [arXiv:1709.09695 [hep-th]].

- Motivated in part by Nico's constant questions about what theories we should pay attention to. Try to encourage engagement on how to decide this and how to deal with such theories.

- (Most of this will be old news to particle theorists.)

Beyond the Standard Cosmological Model

There exist several seemingly distinct ways in which we might explain cosmological observations, including, but not limited to, late-time cosmic acceleration. Beyond a simple cosmological constant, the space of allowed explanations includes

- **Dynamical Dark Energy**: Inflation at the other end of time and energy. Challenging to present a natural model. Requires a solution to CC problem.

- **Modifying Gravity**: Spacetime responds in a new way to the presence of more standard sources of mass-energy. Extremely difficult to write down theoretically well-behaved models, hard to solve even then. But, holds out chance of jointly solving the CC problem. And - opens up the possibility for entirely new tests!

A common Language - EFT

How do particle theorists think about all this? In fact, whether dark energy or modified gravity, ultimately, around a background, it consists of a set of interacting fields in a Lagrangian. The Lagrangian contains 3 types of terms:

- Kinetic Terms: e.g.

$$\partial_\mu\phi\partial^\mu\phi \qquad F_{\mu\nu}F^{\mu\nu} \qquad i\bar{\psi}\gamma^\mu\partial_\mu\psi \qquad h_{\mu\nu}\mathcal{E}^{\mu\nu;\alpha\beta}h_{\alpha\beta} \qquad K(\partial_\mu\phi\partial^\mu\phi)$$

- Self Interactions (a potential)

$$V(\phi) \qquad m^2\phi^2 \qquad \lambda\phi^4 \qquad m\bar{\psi}\psi \qquad m^2h_{\mu\nu}h^{\mu\nu} \qquad m^2{h^\mu}_\mu{h^\nu}_\nu$$

- Interactions with other fields (such as matter, baryonic or dark)

$$\Phi\bar{\psi}\psi \qquad A^\mu A_\mu\Phi^\dagger\Phi \qquad e^{-\beta\phi/M_p}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \qquad ({h^\mu}_\mu)^2\phi^2 \qquad \frac{1}{M_p}\pi{T^\mu}_\mu$$

Depending on the background, such terms might have functions in front of them that depend on time and/or space.

Many of the concerns and ideas of theorists can be expressed in this language: Weak coupling; technical naturalness; unitarity; screening; uv completions; superluminality, … These are serious concerns in constraining proposals.

One Important Example - Ghost-Free Condition

The Kinetic terms in the Lagrangian, around a given background, tell us, in a sense, whether the particles associated with the theory carry positive energy or not.

- Example of the Kinetic Terms:

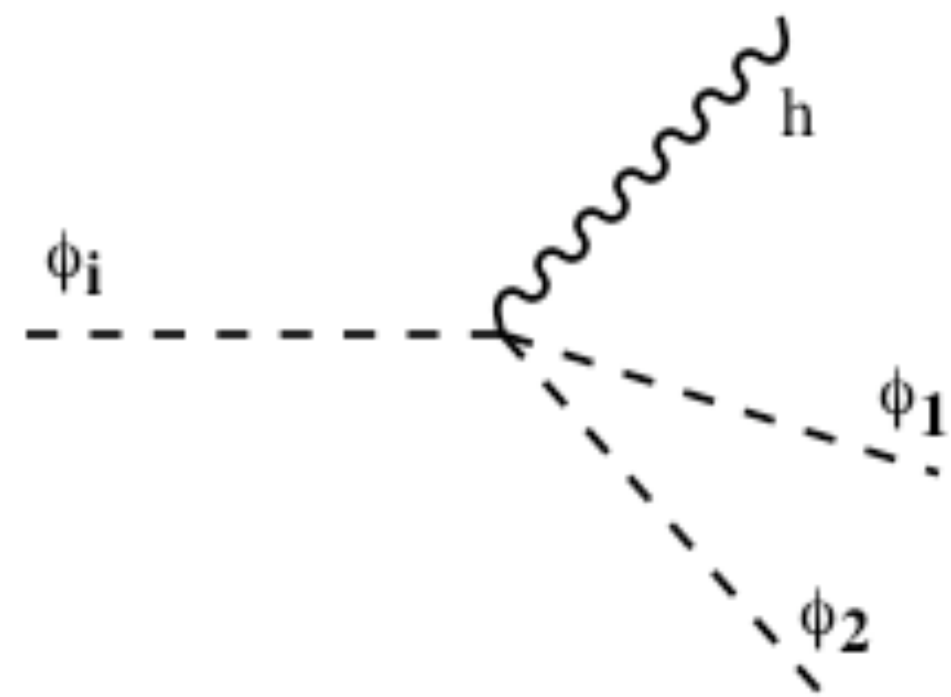
$$-\frac{f(\chi)}{2}K(\partial_\mu\partial^\mu\phi) \to F(t,x)\frac{1}{2}\dot{\phi}^2 - G(t,x)(\nabla\phi)^2$$

This sets the sign of the KE

- If the KE is negative then the theory has ***ghosts***! This can be catastrophic!

If we were to take these seriously, they'd have negative energy!!

- Ordinary particles could decay into heavier particles plus ghosts
- Vacuum could fragment



(Carroll, Hoffman & M.T.,(2003); Cline, Jeon & Moore. (2004))

Most General Second Order Theory

- Motivated by dealing with the ghost problem, much work has focused on **Horndeski Theories:** most general scalar-tensor theory with second-order equations of motion ($X=(\partial\varphi)^2$).

$$
\begin{aligned}
\mathcal{L}_2 &= G_2(\phi, X), \\
\mathcal{L}_3 &= G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)R - 2G_{4,X}\left[(\Box\phi)^2 - \phi_{\mu\nu}^2\right], \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}\left[(\Box\phi)^3 - 3\phi_{\mu\nu}^2\Box\phi + 2\phi_{\mu\nu}^3\right]
\end{aligned}
$$

- Have been applied to **many** different problems in cosmology!
- No particular symmetry principle at work, although subsets of terms represent different theories with different symmetries and motivations - e.g. galileons

Compare with the EFT Approach

- Write down the most general Lagrangian consistent with particle content and symmetries, expanded in powers of E/M (M some scale signaling breakdown of EFT - the cutoff)
- Theoretical constraints apply - locality, analyticity, etc., **BUT** - typically do **not** require second-order equations of motion.
- Instead adopt rules for dealing with higher-derivative operators.
- When should (and shouldn't) we restrict to ghost-free theories like Horndeski?
- What new higher-derivative terms can one write down?
- Are these phenomenologically interesting?

Dealing with Higher Derivatives

- Higher derivatives lead to ghosts, or, in classical language, to new, **spurious** solutions.
- Only a subset of these are **physical** insofar as they reflect solutions of the full theory
- Need to ensure that ghost instabilities associated with higher derivatives are **not present** in physical solutions - rather they should modify the ghost-free solutions.
- Given a higher-derivative EFT, how do we identify these physical solutions? Can't merely solve the equations of motion.
- Are they (exact) solutions to some other, ghost-free theory?
- If yes: justifies use of ghost-free theories
- If no: opens up theory space

A Simple Example

$$S = \int \mathrm{d}^4x \left[-\frac{1}{2}\partial_\mu \Phi^\star \partial^\mu \Phi - V(\Phi^\star \Phi) \right]$$

(Burgess and Williams 1404.2236)

- Writing $$\Phi(x) = \frac{v}{\sqrt{2}} \left(1 + \rho(x)\right) e^{i\theta(x)}$$

  have a massless Goldstone θ and a massive ρ with $M^2=\lambda v^2$

- Action becomes $$\mathcal{L} = \int \mathrm{d}^4x \left[-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho) \right]$$

- For energies << M, we can **integrate out** ρ to obtain an effective action for θ

- Solution to ρ eom is highly nonlocal, but can localize by writing perturbatively in 1/M

$$\Box\rho - (1+\rho)(\partial\theta)^2 - V' = 0 \quad \Longrightarrow \quad \rho = -\frac{(\partial\theta)^2}{M^2} - \frac{(\partial\theta)^4 + 2\Box(\partial\theta)^2}{2M^4} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

When do the physical solutions correspond to an action?

- At low energies, EFT for angular mode θ is

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu\rho}\theta^{,\nu}\theta_{,\rho} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

The key is **field redefinitions**: map

$$\theta \to \theta + \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu}\theta^{,\nu}$$

- This leaves us with a second-order theory, the quartic galileon!

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 + \frac{1}{M^4}(\partial\theta)^2\left[\theta_{,\mu\nu}\theta^{,\mu\nu} - (\Box\theta)^2\right] + \mathcal{O}\left(\frac{1}{M^6}\right)$$

- The problematic higher derivatives have been shunted off to $O(M^{-6})$, which we can safely ignore

- Physical solutions to this EFT could be obtained by exactly solving a quartic galileon (up to $O(M^{-4})$)

How to Identify Genuine Higher Derivatives

- Construct EFT operator basis up to terms equivalent via equations of motion. (Extra ingredient for modified gravity: construct bases with as few higher derivatives as possible)
- Those higher-derivative terms that are left should be included!
- Deal with these by solving equations of motion perturbatively

- e.g. EFT of shift-sim scalar.
- Note: no ghosts until dimension 12!
- Related story (with different results), for scalar-tensor.

Dimension	Operators
4	$X = (\partial\varphi)^2$
5	*None*
6	*None*
7	*None*
8	X^2
9	*None*
10	Quartic galileon
11	*None*
12	$X^3, (\varphi_{,\mu\nu}\varphi^{,\mu\nu})^2$

e.g. EFT Basis for Modified Gravity

- Consider scalar-tensor EFT in derivative expansion

Derivatives	Operators
4	X^2, Gauss-Bonnet [Weinberg - EFT of Inflation]
6	X^3, quartic Horndeski, *five new higher-derivative operators*

- Consider six-derivative higher-deriv operators alongside comparable-size Horndeski terms in, e.g., inflation, dark energy. *No reason* a priori to ignore them

$$R_{\mu\nu\alpha\beta}\phi^{;\mu\alpha}\phi^{;\nu\beta}, \quad R_{\mu\nu\alpha\beta}\phi^{;\mu}\phi^{;\alpha}\phi^{;\nu\beta}, \quad XR^2_{\mu\nu\alpha\beta},$$
$$R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu}, \quad (\nabla R_{\mu\nu\alpha\beta})^2$$

- Must deal with higher derivatives either by solving eoms order by order; or by reducing order of eoms using perturbative nature

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Main Message

- When thinking about testing, think of modified gravity as an EFT, including allowing higher derivatives.

- Lesson from particle physics - a healthy UV theory does not imply ghost-free low-energy EFT. However, associated Ostrogradski ghost can be an artifact of the EFT truncation and does not lead to a physical instability (well-known). Nevertheless presents a challenge when solving equations numerically,

- Additional justification is required to restrict to theories with second-order equations of motion. Therefore, e.g., both Horndeski *and non-Horndeski* terms should be treated in an EFT expansion.

- Techniques discussed here used to redefine terms etc., allow us to know when we can define away such terms, when they may be described by another theory, with only second order eoms, or when they need to be dealt with in some other way.

Thank You!