

# EMRI modeling and systematics

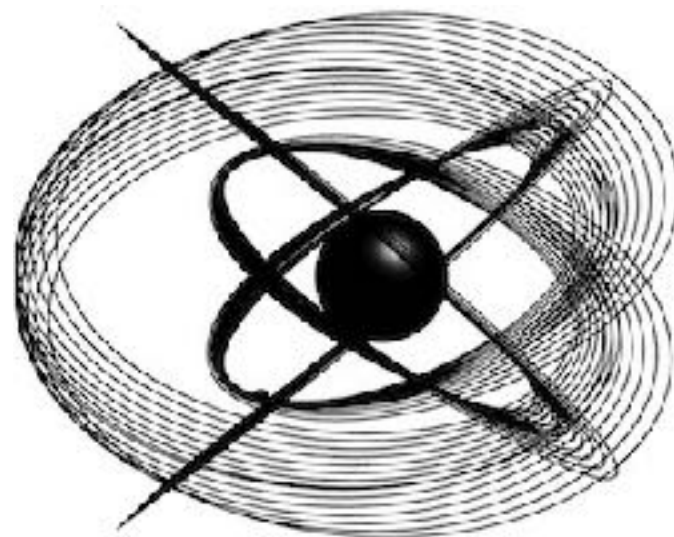
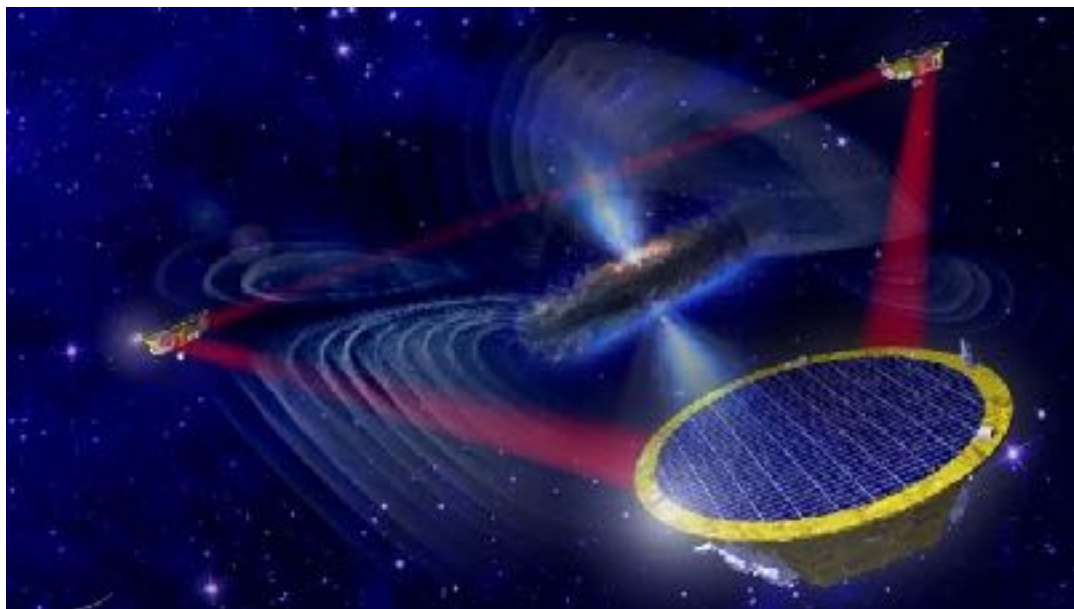
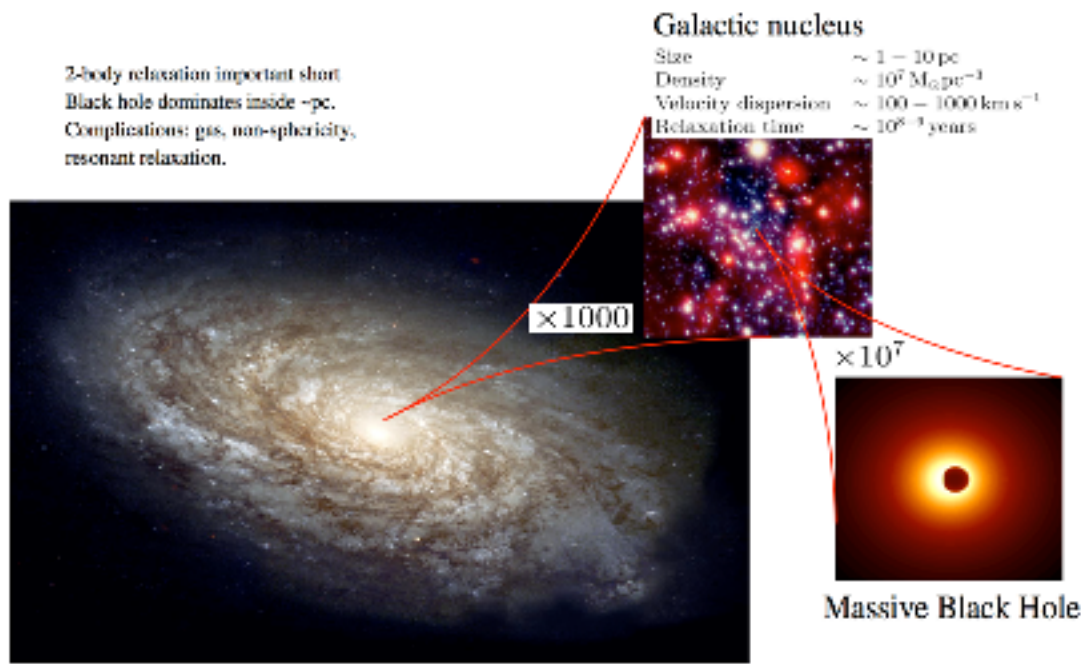


Image: Steve Drasco, CalPoly

What modeling and data analysis work must be done in order to achieve the science that has been promised for extreme mass ratio inspiral measurements?

# Astrophysics overview

The setting: Center of a “normal” galaxy. Typically hosts a black hole of  $10^6$ – $10^7$  solar masses; black hole in a nucleus with  $\sim 10^9$  solar masses of stars.

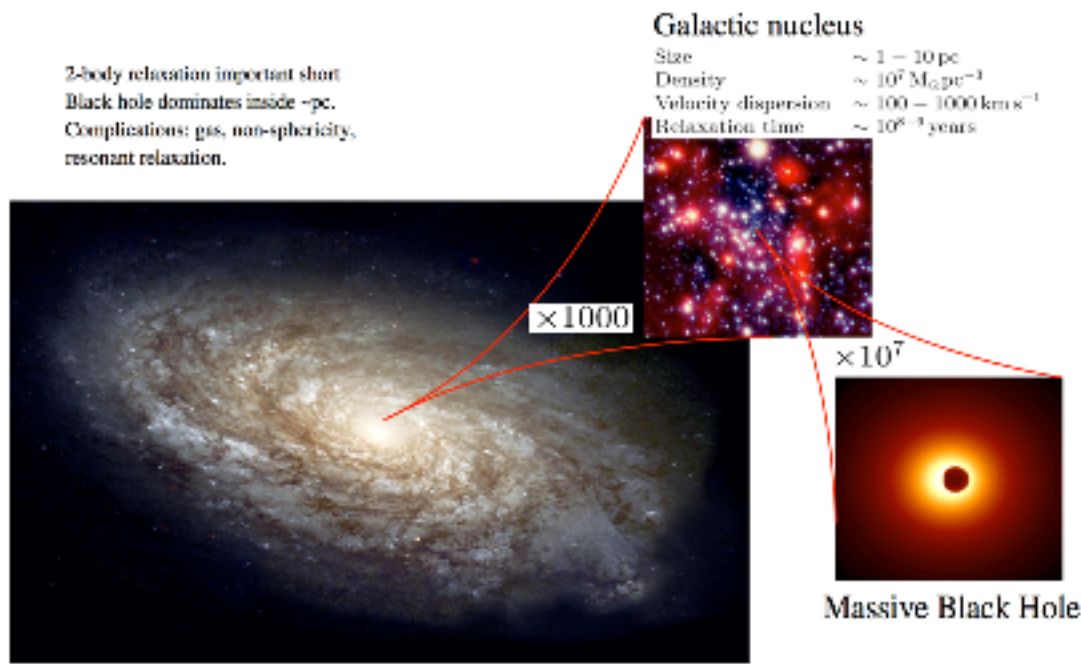


Graphic courtesy of Marc Freitag

The most massive of these stars tend to sink closest to the large black hole; these stars evolve through main sequence most quickly, will leave stellar mass black holes behind.

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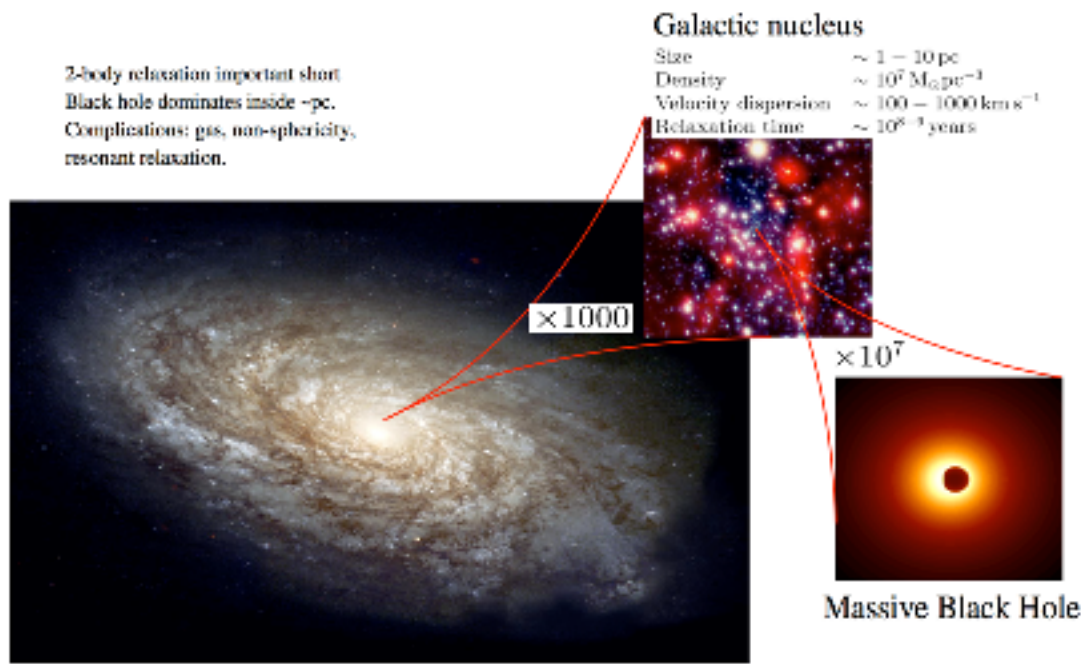


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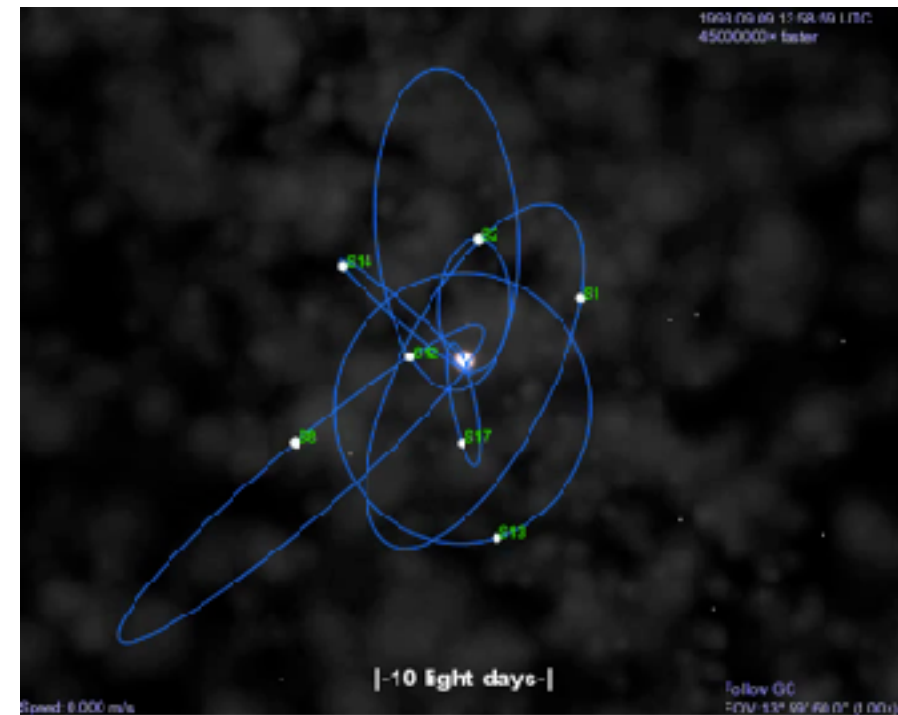
Multi-body scattering in centers of galaxies puts compact stellar remnant onto an orbit that evolves into a strong-field, GW-driven inspiral.

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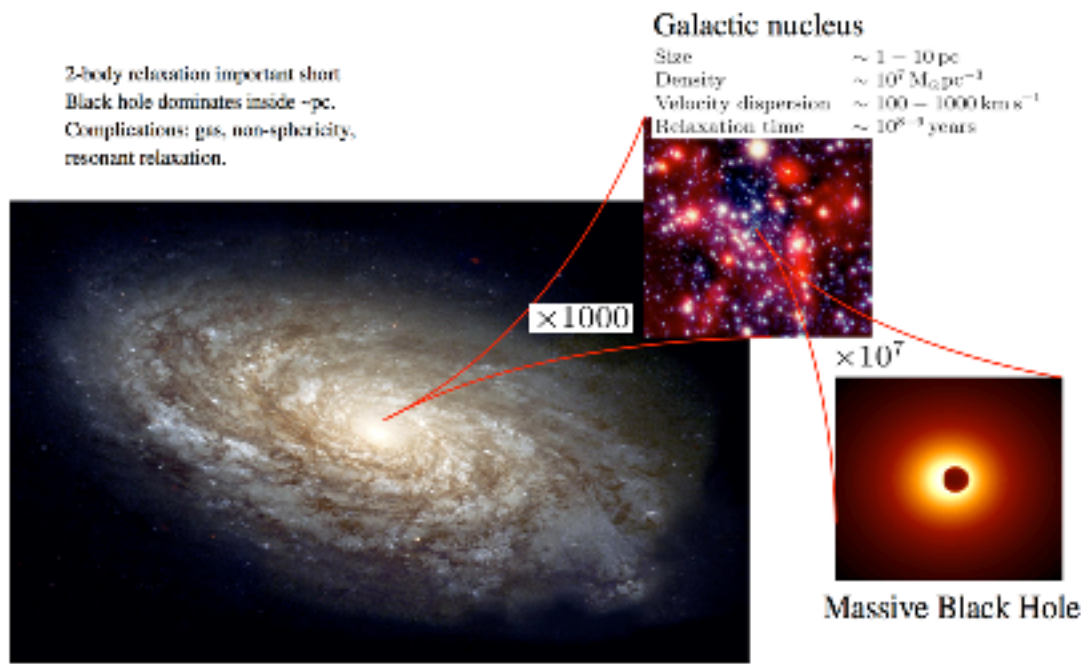
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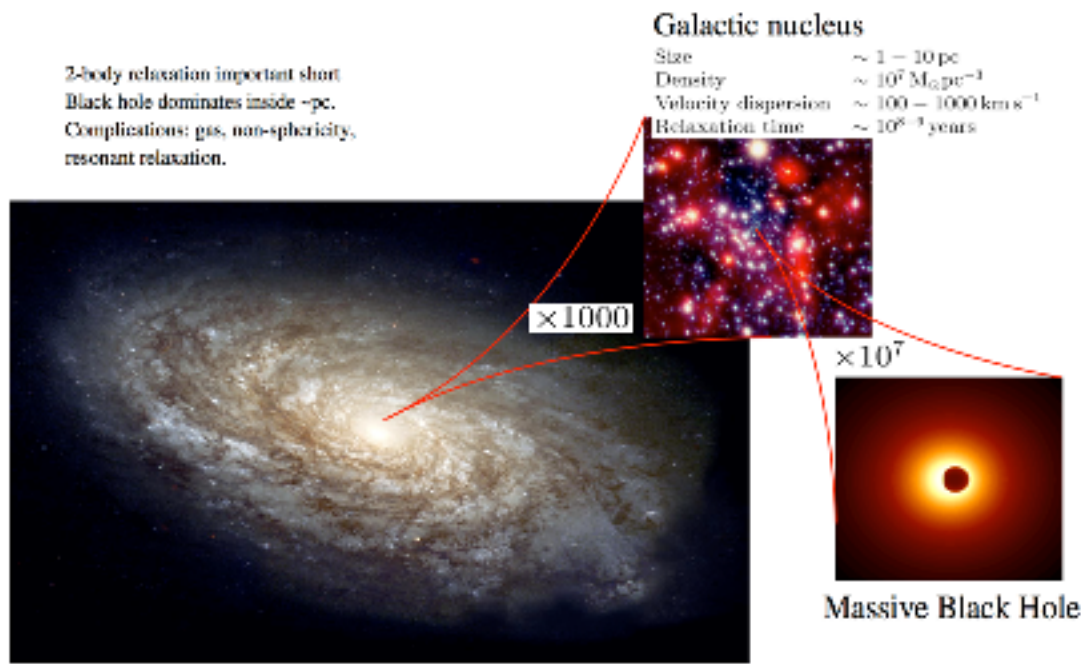
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Gravitational waves generated by these extreme mass ratio inspirals are in band

$$\frac{c^3}{50GM} \lesssim f \lesssim \frac{c^3}{GM}$$

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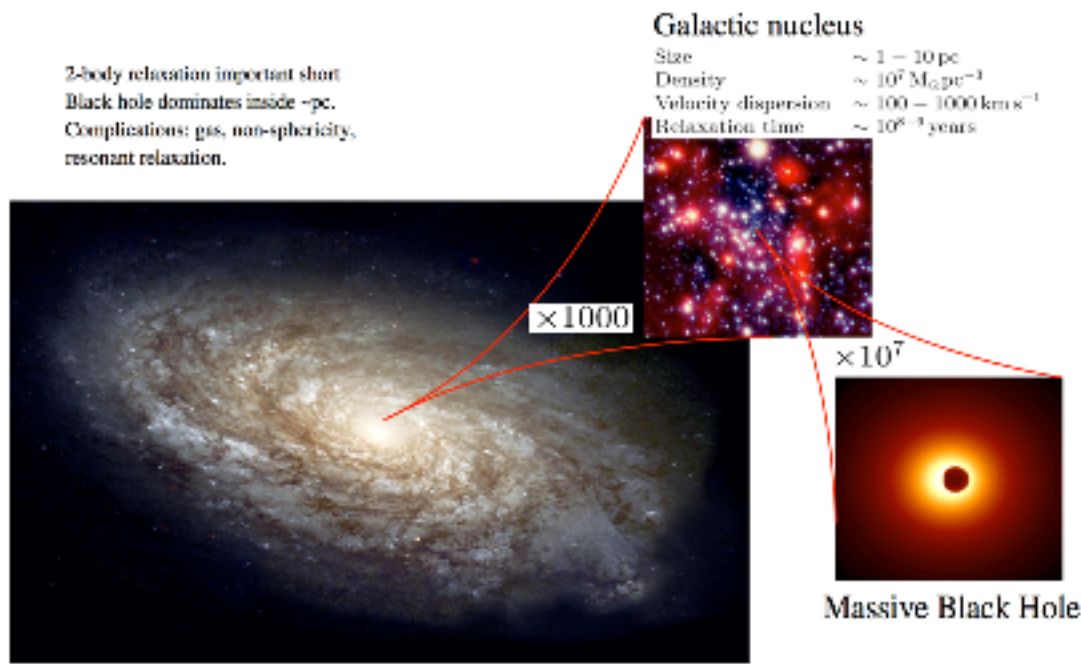
$$0.004 \text{ Hz} \lesssim f \lesssim 0.2 \text{ Hz}$$

at  $10^6 M_{\text{sun}}$

Perfect for LISA!

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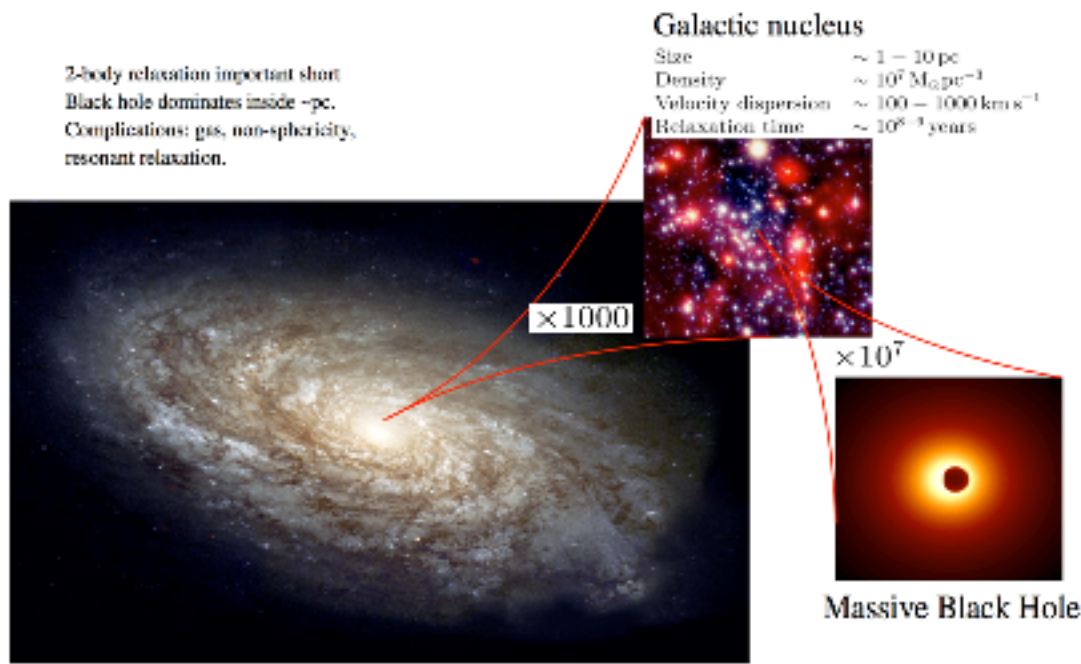


Graphic courtesy of Marc Freitag

Number of galaxies with the “right” central BHs plus studies of stellar scattering processes indicate event rate likely to be high: Perhaps  $> 10^2$  per year if smaller object is 30  $M_{\text{sun}}$  or larger.

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Graphic courtesy of Marc Freitag

Interesting related case:  
**INTERMEDIATE** mass ratio  
inspiral or IMRI. If smaller  
body smaller body is  $10^2$   
or  $10^3 M_{\text{sun}}$  black hole,  
events are detectable to  
high redshift. **Likely an  
important fraction of  
early black hole mergers!**



# The physics view of an EMRI

Get some intuition by using leading order formulas:

Time spent spiraling from  $f = f_1$  to  $f = f_2$ :

$$T_{\text{band}} = \frac{5}{2^{2/3} \pi^{8/3} 1024} \frac{c^3}{G\mu} \frac{c^2}{(GM)^{2/3}} \left( f_1^{-8/3} - f_2^{-8/3} \right)$$

Months to years in band for  $M \sim 10^6 - 10^7 \text{ Msun}$ ,  
 $\mu \sim 5 - 50 \text{ Msun}$ .

Number of orbits executed in that time:

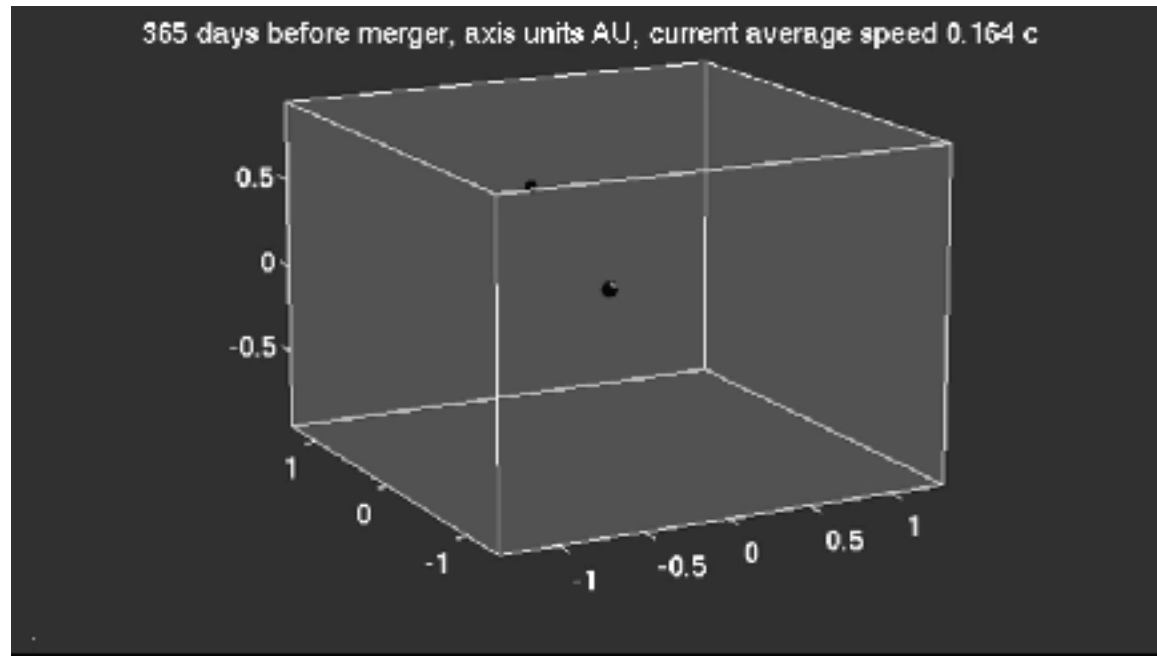
$$N_{\text{orb}} = \frac{1}{2^{2/3} \pi^{8/3} 256} \frac{c^3}{G\mu} \frac{c^2}{(GM)^{2/3}} \left( f_1^{-5/3} - f_2^{-5/3} \right)$$

Tens of thousands of orbits executed during  
that time in band for these masses.

# The physics view of an EMRI

Thens of thousands of slowly evolving orbits are executed in the near-field region of large black hole's spacetime ... GWs that they generate are sensitive to the near-horizon black hole spacetime.

If we can coherently track these GWs, can use them to measure spacetime properties; expect measurement errors to scale as  $1/N_{\text{orb}}$  and  $1/(\text{signal to noise})$ .



# Probe of BH spacetime

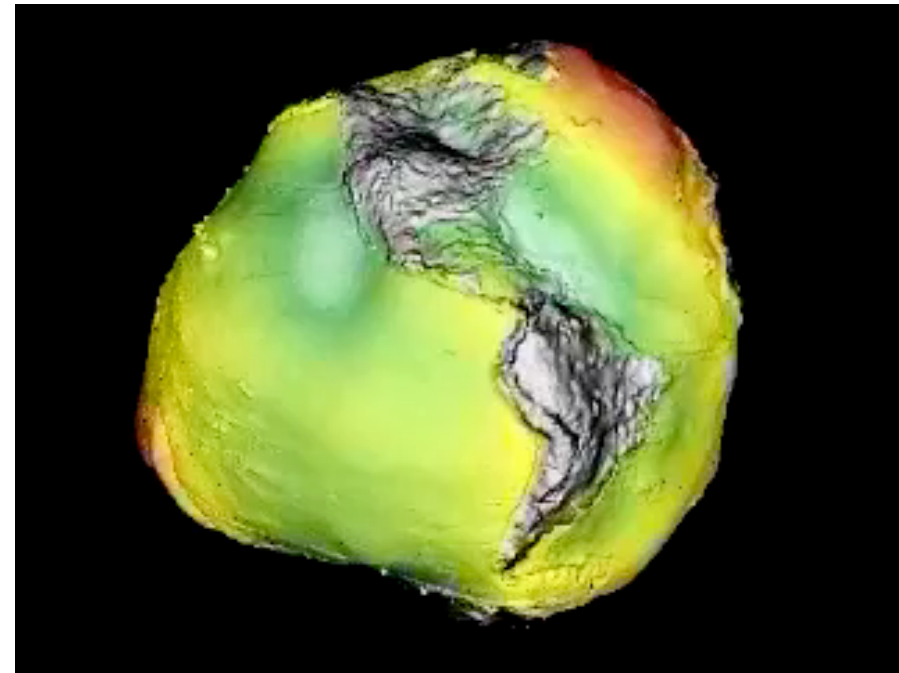
Thanks to large mass ratio, EMRIs function as *nearly* a test particle probe of black hole spacetimes.

Measurement analogous to geodesy: Measurements of orbit precisely map gravitational potential; enforce field equation

$$\nabla^2 \Phi_g = 4\pi G \rho_m$$

infer mass distribution.

$$\Phi_g = -\frac{GM}{r} + \sum_{l=2}^{\infty} \sum_{m=-l}^l M_{lm} Y_{lm}(\theta, \phi)$$



GRACE gravity model

# Probe of BH spacetime

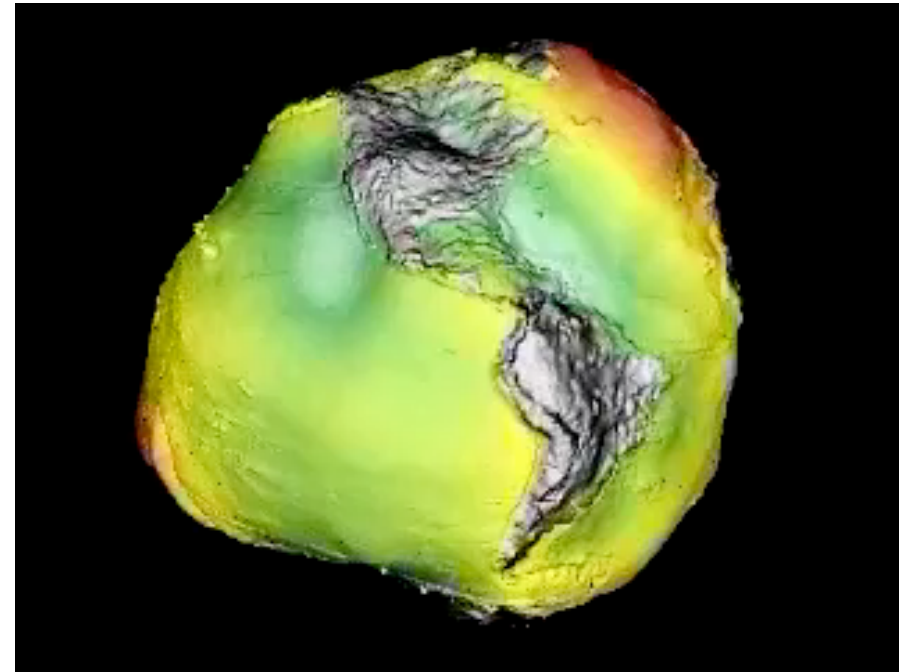
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**Bothrodesy:** Mapping the multipoles that govern a black hole spacetime.

Kerr expectation: Axisymmetry  
(no non-zero  $m$  modes)

Mass and current moments  
set by hole's mass and spin:

$$M_l + iS_l = M(ia)^l$$



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**Mass, spin, mass ratio:**

$$\delta M/M, \delta a, \delta \eta \sim 10^{-5} - 10^{-3}$$

**Orbit geometry:**

$$\delta e \sim 10^{-5} - 10^{-3}$$

$$\delta(\text{spin direction}) \sim \text{a few deg}^2$$

$$\delta(\text{sky position}) \lesssim 10 \text{ deg}^2$$

**Distance to binary:**

$$\delta D/D \sim 0.03 - 0.1$$

Barack & Cutler PRD **69**, 082005 (2004)

Babak et al PRD **95**, 103012 (2017)

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**Higher multipoles:**

$$\delta(\text{mass quadrupole}) \sim 10^{-3}$$

[degrades by  $\sim 3$  each increase in  $l$ ]

Ryan PRD **56**, 1845 (1997); Barack & Cutler PRD **75**, 042003 (2007)

Babak et al PRD **95**, 103012 (2017)

# Can we deliver this science?

Need modeling technique that allows us to compute GWs arising from the fast motion, near-horizon region that characterizes EMRIs.

Post Newtonian: Expansion in  $(v/c) \sim (GM/rc^2)^{1/2}$ , not good for fast orbital speeds, small orbital radius.

Numerical relativity: Need to resolve spacetime curvature on length scale comparable to the small body's Schwarzschild radius for tens of thousands of orbits.

**Black hole perturbation theory:** Treat binary as the exact black hole solution of the larger body, with a perturbation arising from the orbiting companion.

Mass ratio defines a perturbative expansion.

# Black hole perturbation theory

Schematically, write the spacetime as

$$g_{\alpha\beta} = g_{\alpha\beta}^{\text{Kerr}}(M, a) + h_{\alpha\beta}^{(1)} + h_{\alpha\beta}^{(2)} + \dots$$

Compute corrections order by order in system's mass ratio.

Motion of small body looks like a geodesic of  $\mathbf{g}^{\text{Kerr}}$  plus corrections that arise from perturbations  $\mathbf{h}^{(n)}$ :

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = f^\alpha$$

“Self force” – correction to geodesic black hole orbits due to  $\mathbf{h}$  terms.

Comment: Treating secondary as a test mass; imagining that this binary is in an otherwise empty universe.

Small body will actually have structure (e.g., spin), and the universe contains other objects ... return to these points later.



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- \* **Dissipative self force:** Takes energy and angular momentum from binary, drives long-time evolution of orbit.
- \* **Conservative self force:** Conserves energy and angular momentum, shifts away from geodesic (e.g., changing orbit frequencies).

# Counting phase accumulation

Conservative and dissipative effects contribute to accumulated phase at different orders in mass ratio:

Phase accumulated  
from  $t_1$  to  $t_2$ :

$$\Phi(t_1, t_2) = \int_{t_1}^{t_2} \omega(t) dt$$

$O(M/m)$ : Evolving geodesic frequency  
[ $O(1/M)$ ] integrated over inspiral [ $O(M^2/m)$ ]  $\longrightarrow = \Phi_{\text{diss}-1}$

$O(1)$ : Conservative correction to frequency  
[ $O(m/M^2)$ ] integrated over inspiral  $\longrightarrow + \Phi_{\text{cons}-1}$

$O(1)$ : Geodesic frequency integrated against  
next correction to inspiral [ $O(M)$ ]  $\longrightarrow + \Phi_{\text{diss}-2}$

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# Counting phase accumulation

**Conventional wisdom:** Detecting GWs requires models accurate to  $O(1)$  in phase.

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# Counting phase accumulation

**Conventional wisdom:** Best GW science fits needs accuracy  $O(1/\text{SNR})$  in phase.

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		(and beyond)



# Where does modeling stand today?

For *detection only*, arguments based on phase counting and conventional wisdom tell us

$$\Phi_{\text{needed}} = \Phi_{\text{diss}-1} + \Phi_{\text{cons}-1} + \Phi_{\text{diss}-2}$$

From 1st order averaged  
dissipative self force.

***Totally understood;  
expensive to compute***

From 1st order averaged cons. & 1st  
order oscillatory diss. self force.

***Understood; extremely  
expensive & challenging  
to calculate.***

From 2nd order  
averaged dissipative  
self force:

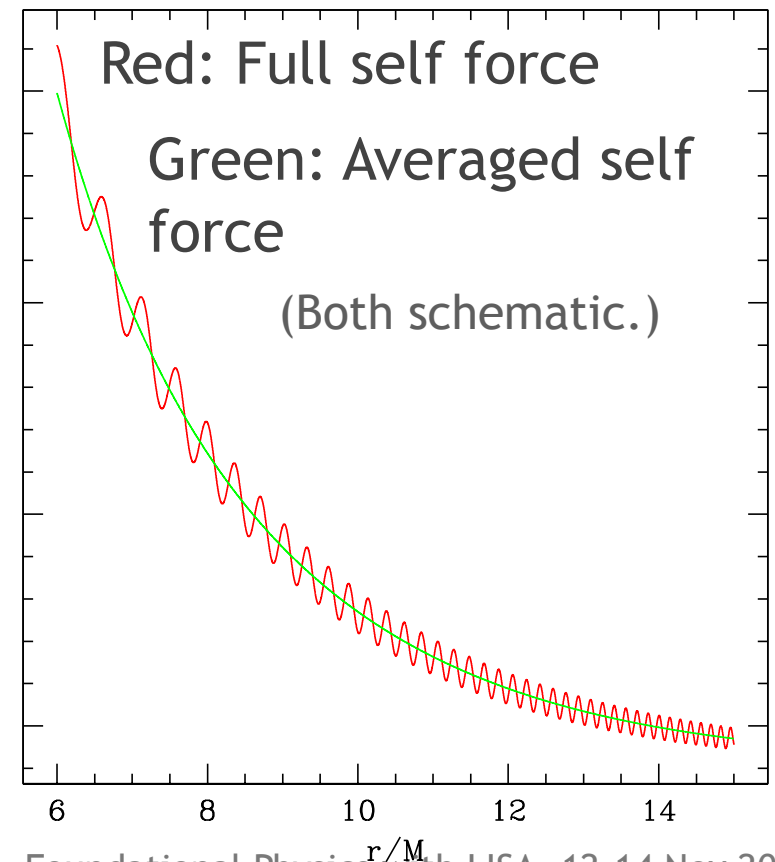
***Current frontier:  
first results  
(almost) in hand.***

# Surprise! Conventional wisdom breaks down: Averaging is not so simple.

As this picture was developing, Flanagan and Hinderer [PRD 78, 064028 (2008)] found Kerr black hole orbits can “break” the averaging underlying this analysis.

Self force can be split into “average” and “oscillatory” pieces:

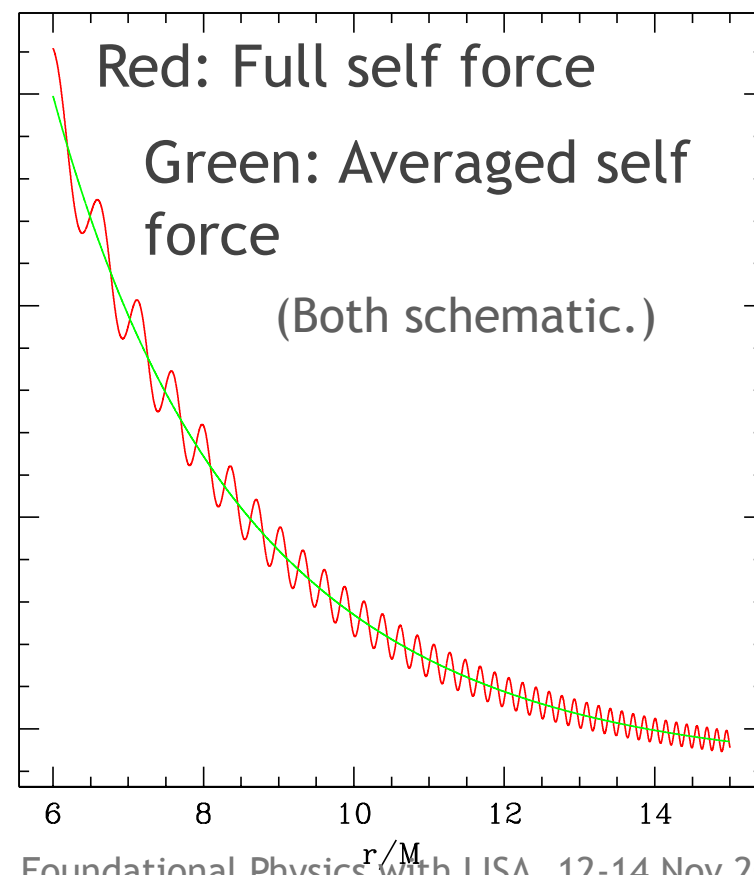
$$\begin{aligned} f^\gamma &= \sum_{kn} (f^\gamma)_{kn} e^{-i(k\Omega_\theta + n\Omega_r)t} \\ &= (f^\gamma)_{00} + \sum_{\substack{kn \\ k \neq 0 \ n \neq 0}} (f^\gamma)_{kn} e^{-i(k\Omega_\theta + n\Omega_r)t} \end{aligned}$$



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For “most” orbits, the oscillatory contribution is much less important than the average ... can neglect at leading order, use only the average components.

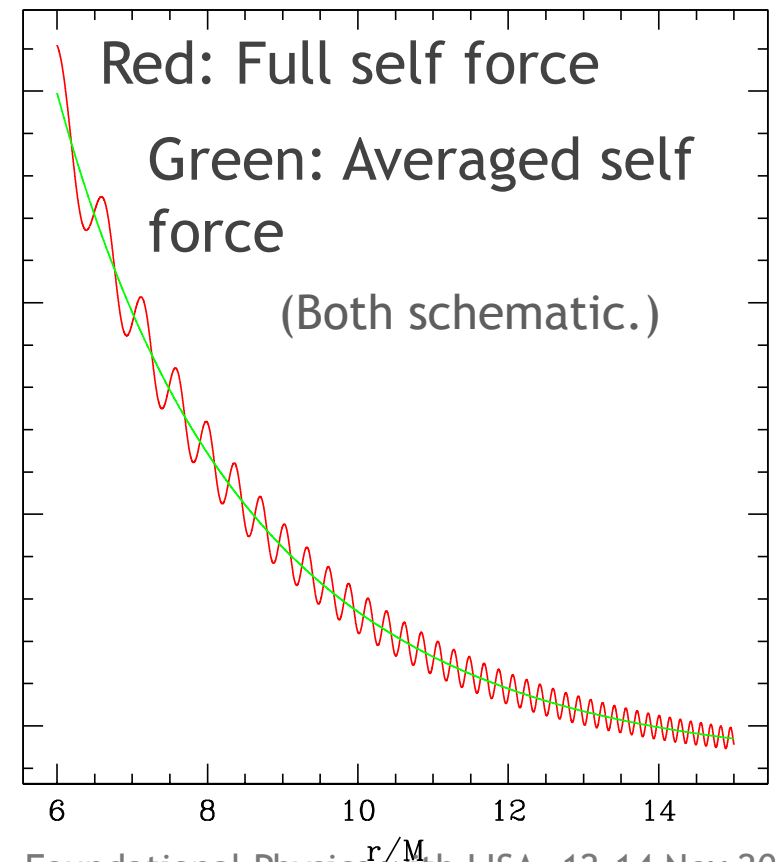


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**BUT:** There exist “resonant” orbits for which “oscillatory” piece doesn’t oscillate.

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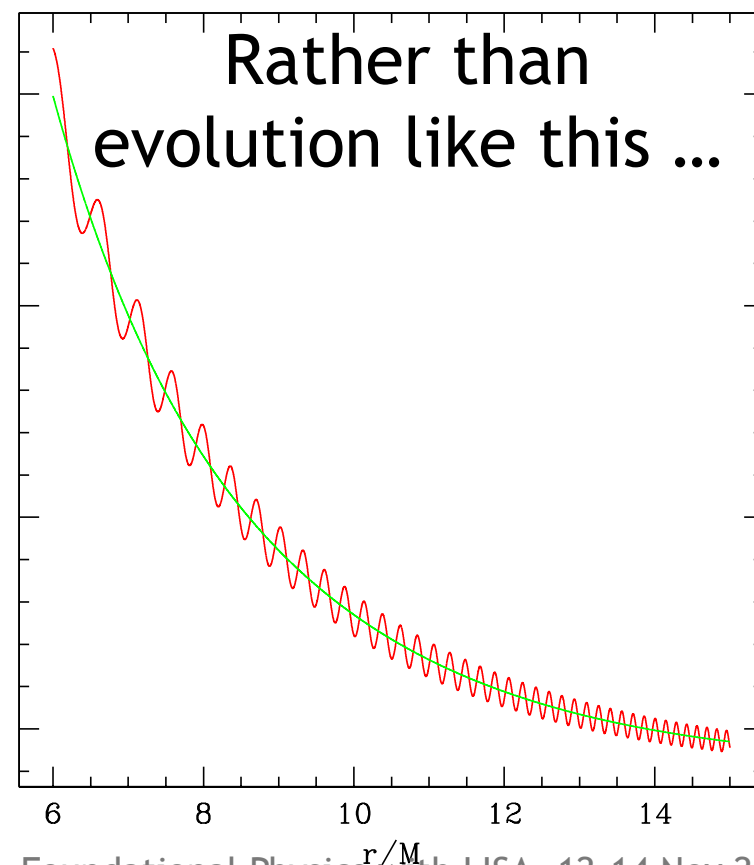
Example: When  $\Omega_\theta = 2\Omega_r$ , the “oscillating” term is constant for all terms in the sum in which  $n = -2k$ .

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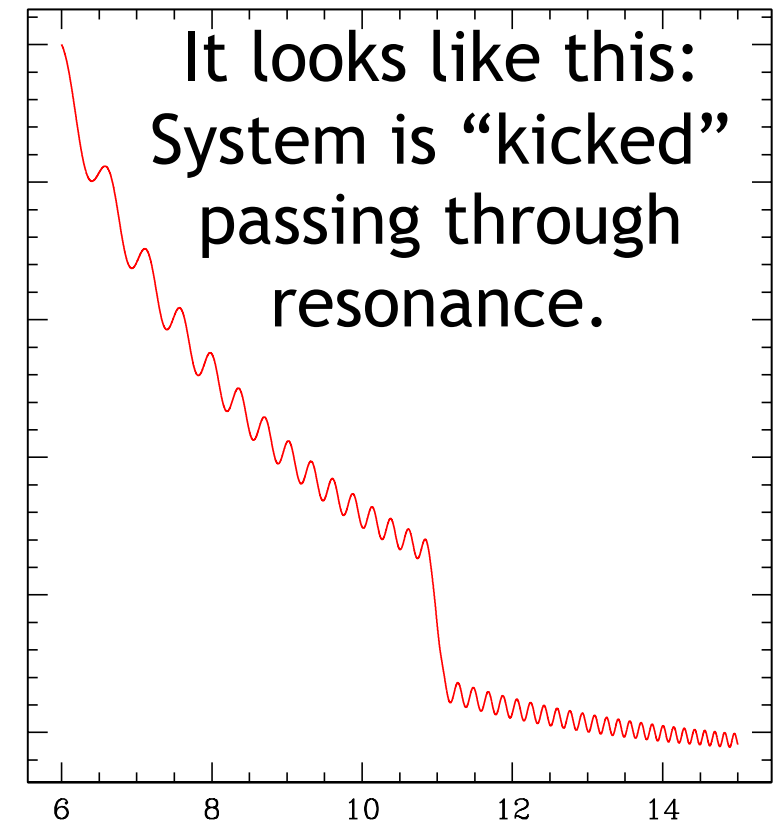


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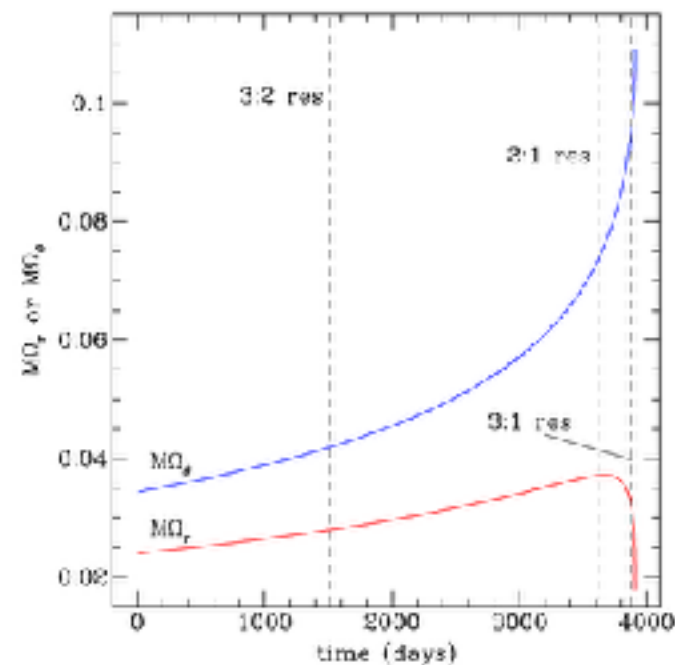
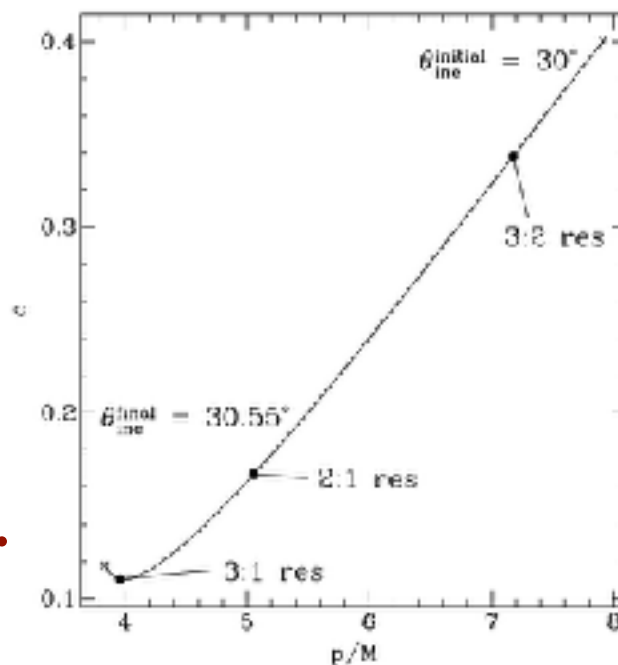
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This behavior is **generic**: *Every* EMRI encounters at least one resonance as it spirals through the strong-field. Many encounter two; a few encounter three.

Example: An inspiral that encounters three resonances in its last ~8 years of inspiral ... two of them in final 250 days before plunge.



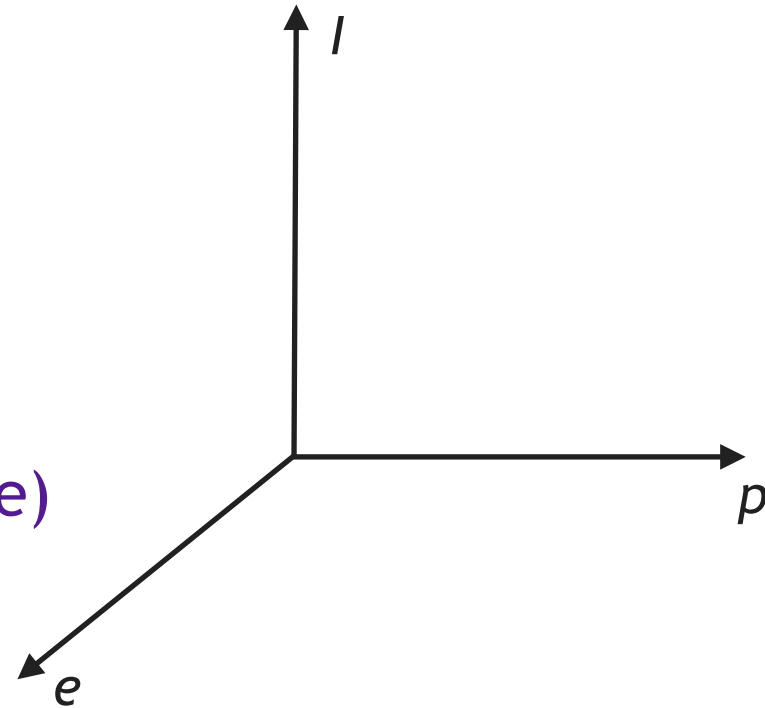
[1  $M_{\text{Sun}}$  spiraling into  $10^6 M_{\text{Sun}}$  with spin  $a/M = 0.7$ . From Ruangsri and Hughes, PRD **89**, 084036 (2014).]

# Computational cost

Major issue: Significant computational cost to cover the parameter space and make models.

## Recipe for simplest model:

1. Lay out grid in orbit parameter space ( $10^3 - 10^4$  points to cover strong field)
2. Solve linearized Einstein equation at each point ( $10^2 - 10^4$  multipoles per point ... about 0.01–0.1 CPU seconds per multipole)
3. Use data to evolve from orbit to orbit, build waveform.



Good news: Steps 1 and 2 need only be done ***once***.  
When those data exist, can be stored, waveform computed (fairly) quickly each time.

# Kludges<sup>1</sup>: Approximate models for testing EMRI data analysis

Long known that relativistic wave models would be too expensive for EMRI data analysis studies ...  
“kludges” developed as tools for this.



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<sup>1</sup> From Oxford English Dictionary:

Kludge: A hastily improvised and poorly thought-out solution to a fault or ‘bug.’

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*Analytic kludge* of Barack and Cutler (2004): Analytic model based on post-Newtonian approximation to EMRIs. Has main features (3 orbital frequencies, strong precession).

Very fast, very easy to implement. Useful for studying time-frequency structure of EMRI waves in simulated data.

Does not remain phase locked with relativistic models for long time! Great tool for exploring algorithmics, but not accurate model of Nature's EMRIs.

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*Analytic kludge* of Barack and Cutler (2004): Analytic model based on post-Newtonian approximation to EMRIs. Has main features (3 orbital frequencies, strong precession).

**The analytic kludge has been the foundation of nearly all EMRI science studies that have been done to date.**

# Kludges<sup>1</sup>: Approximate models for testing EMRI data analysis

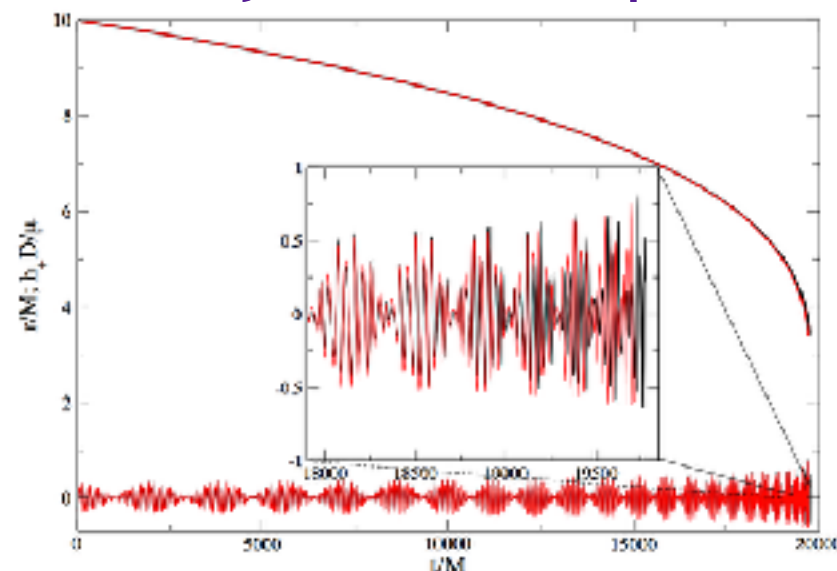
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“kludges” developed as tools for this.

*Numerical kludge* of Babak et al (2008): Use fits to relativistic data for the small body’s inspiral; use a simple multipole formula to make GWs from a small body on that inspiral.

***Much*** slower than the analytic kludge ... but maintains high fidelity with relativistic models.

[From Babak et al, PRD 75, 024005 (2007).]



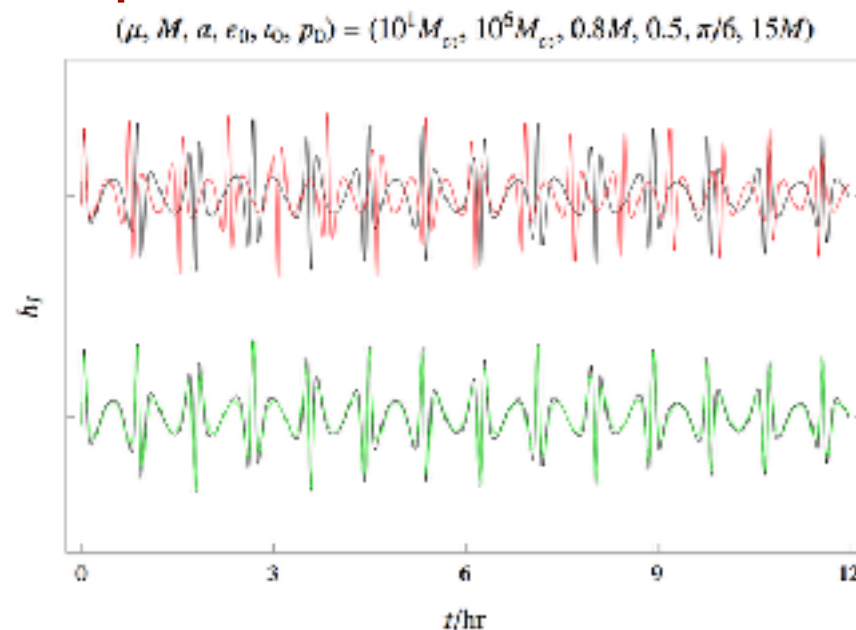
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**Middle ground:** Chua and Gair (2015) greatly improves analytic kludge with little extra computational cost.

Model follows Barack and Cutler recipe for inspiral, but uses *exact* Kerr frequencies at each moment to build waves.

[From Chua and Gair, CQG 32, 232002 (2015).]



# Synopsis and assessment

- \* We fully understand how to compute the leading model for EMRI GWs, at least in the framework of an isolated binary in GR. Much of the calculation is slow, but only needs to be done once.
- \* **All science studies to date based on simplest EMRI models!** Likely to be indicative of “real” results, but must be concerned about details.
- \* We have an urgent need to develop waveforms based on rigorous framework (as few approximations as possible), and to develop extensions to properly assess how well we can test black hole spacetimes and theories of gravity.

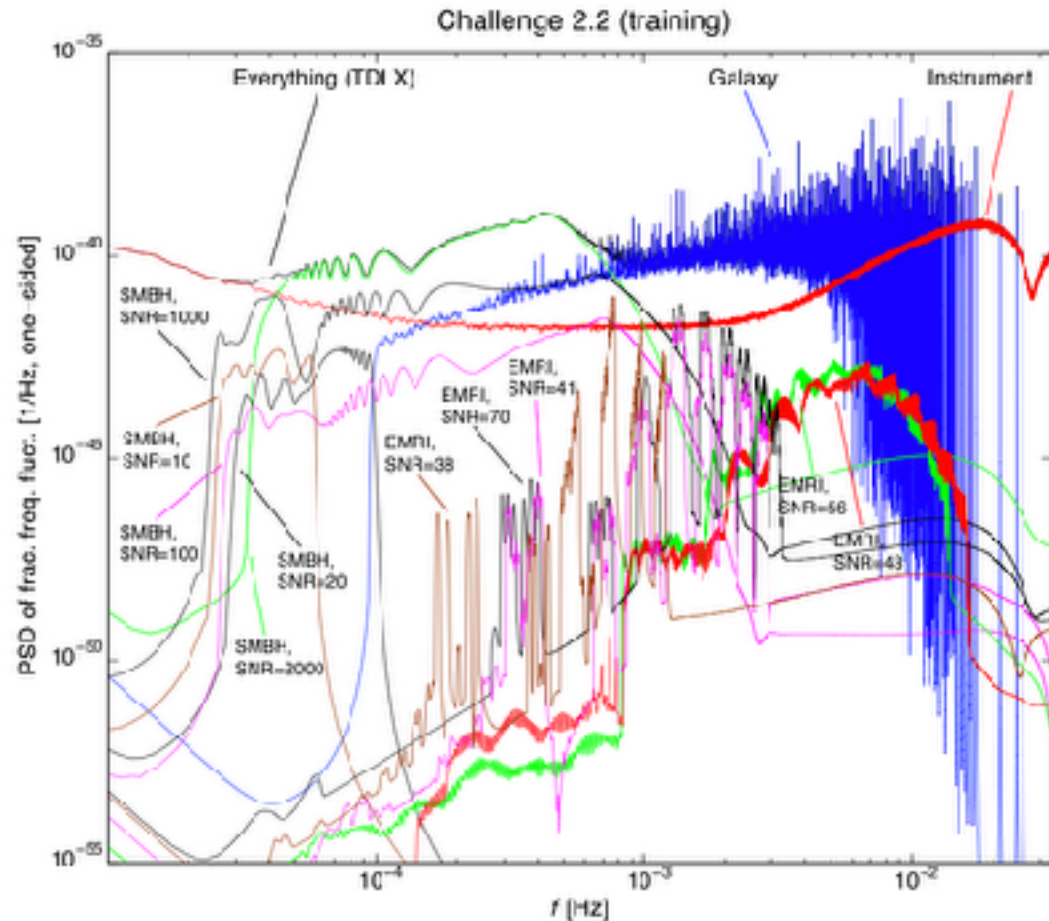


# Top level concern

## EMRI waves are weak waves

EMRI waves are lower in instantaneous amplitude than the mean expected noise level, and that of most sources that are “on” at any moment.

To measure, we must coherently follow phase, “integrate up” signal above background.

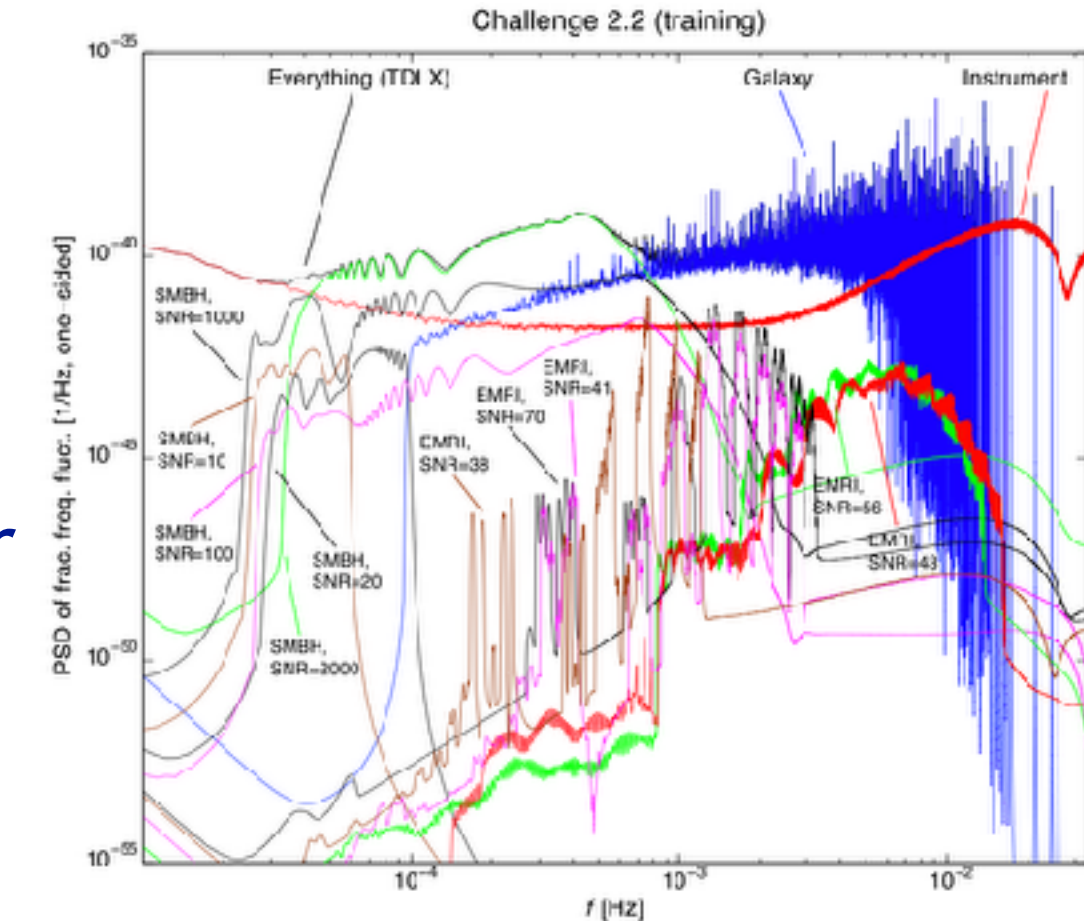


[From Arnaud et al, CQG 24, S551 (2007).]

# Top level concern

## EMRI waves are weak waves

Already looking for a weak signal ... subtle effects (like deviations from Kerr spacetime) may be masked by other subtle effects that we know will be present unless they can be modeled accurately.



[From Arnaud et al, CQG 24, S551 (2007).]

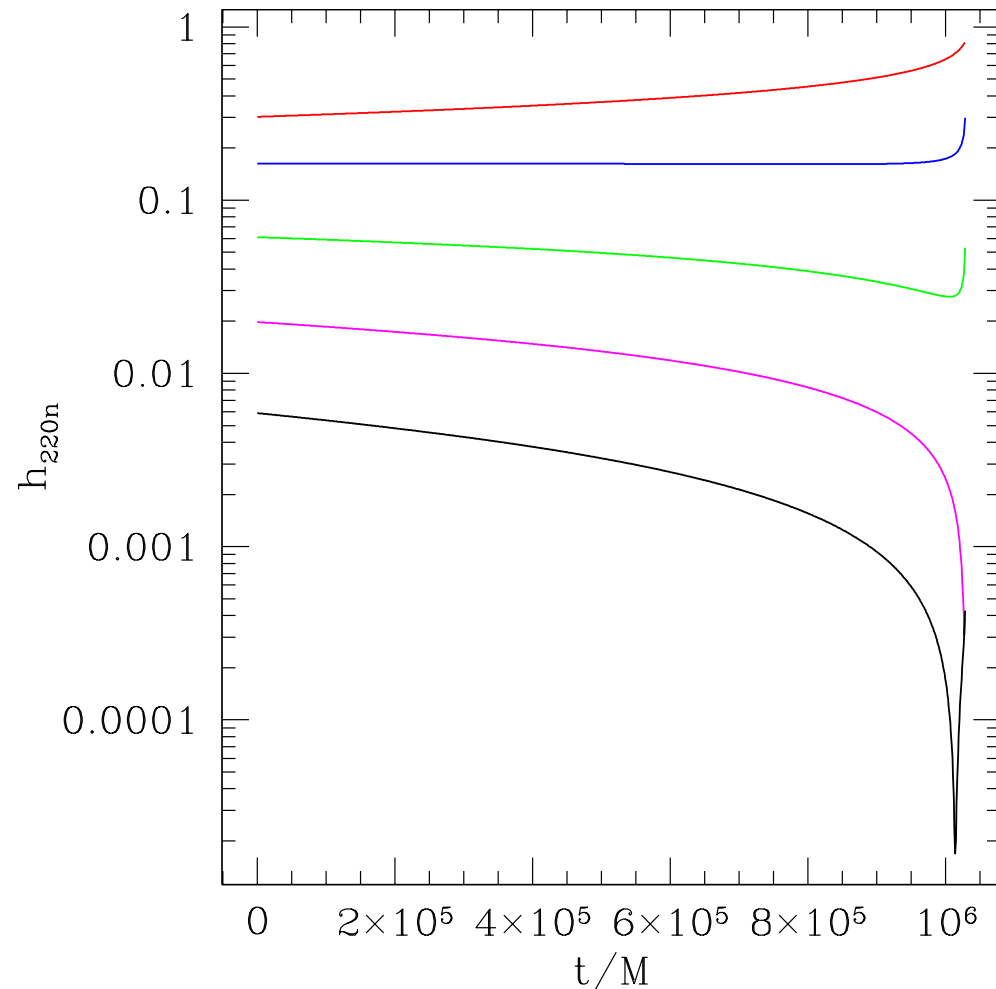
# Examples of issues

EMRI waveforms have a “multivoice” structure:

$$h_+ - ih_\times = \sum_{lmkn} h_{lmkn}(t) e^{i\Phi_{lmkn}(t)}$$

Example: Some  $l = 2, m = 2, n > 0$  voices for inspiral into Schwarzschild (initial  $e = 0.2$ , initial  $p = 16M$ ).

Each voice encodes nature of spacetime: Structure increases leverage on science return, but must be modeled well to avoid bias.



# Examples of issues

Absorption: Back reactive evolution driven by radiation to scri+ and radiation down the horizon

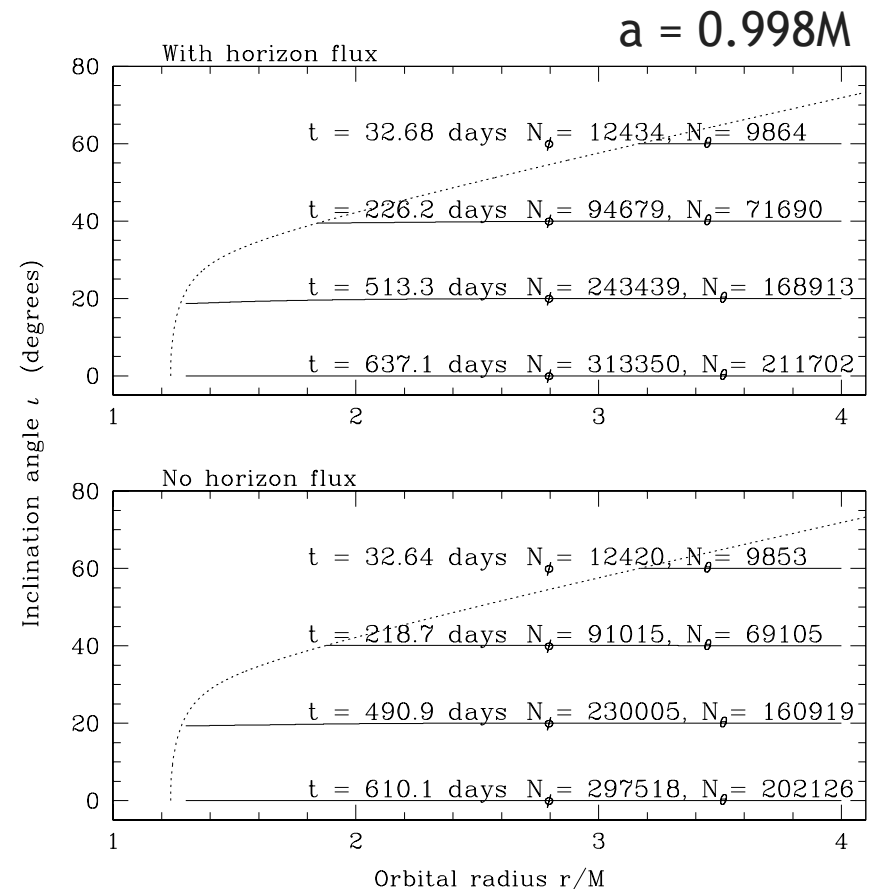
$$\left(\frac{dE}{dt}\right)^{\text{orb}} = -\left(\frac{dE}{dt}\right)^{\infty} - \left(\frac{dE}{dt}\right)^{\text{H}}$$

Horizon term **strongly** depends on black hole spin:

If black hole spins rapidly, absorption can significantly prolong inspiral.

Executes  $\sim 10^4$  additional orbits vs model that does not include absorption.

[From Hughes, PRD 64, 064004 (2001).]



# Examples of issues

Absorption: Back reactive evolution driven by radiation to scri+ and radiation down the horizon

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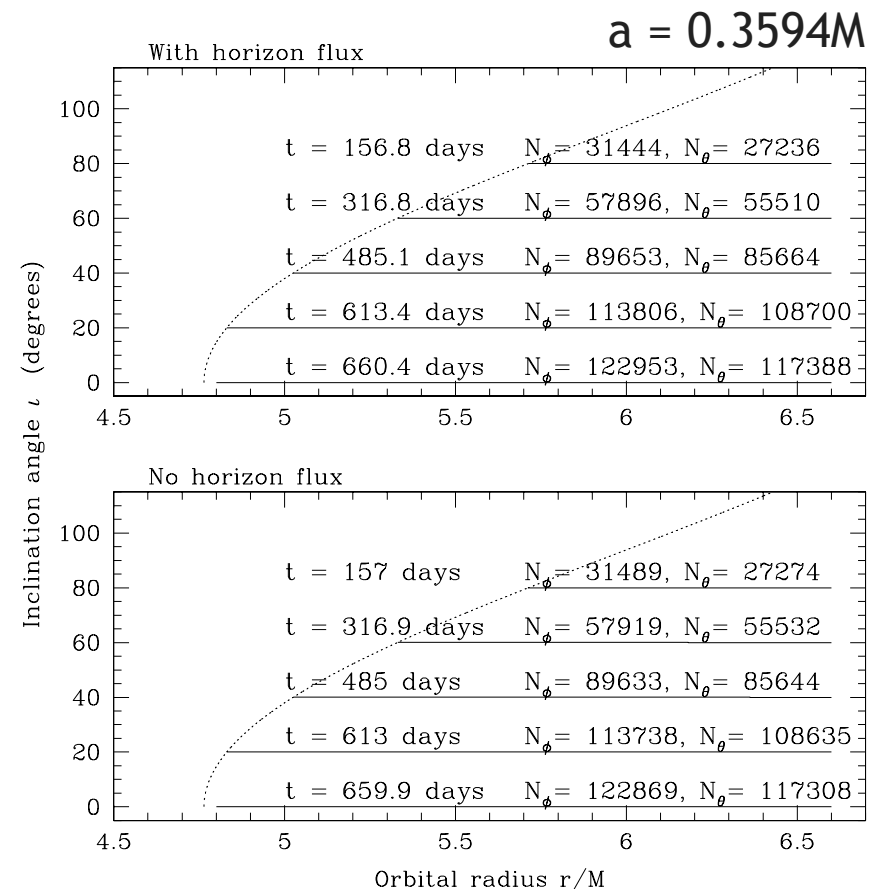
Horizon term **strongly** depends on black hole spin:

For slower spin, effect can be negligible, can switch sign.

Test horizon absorption (e.g., QG structure)?

Beware of correlations with “vanilla” effects.

[From Hughes, PRD 64, 064004 (2001).]

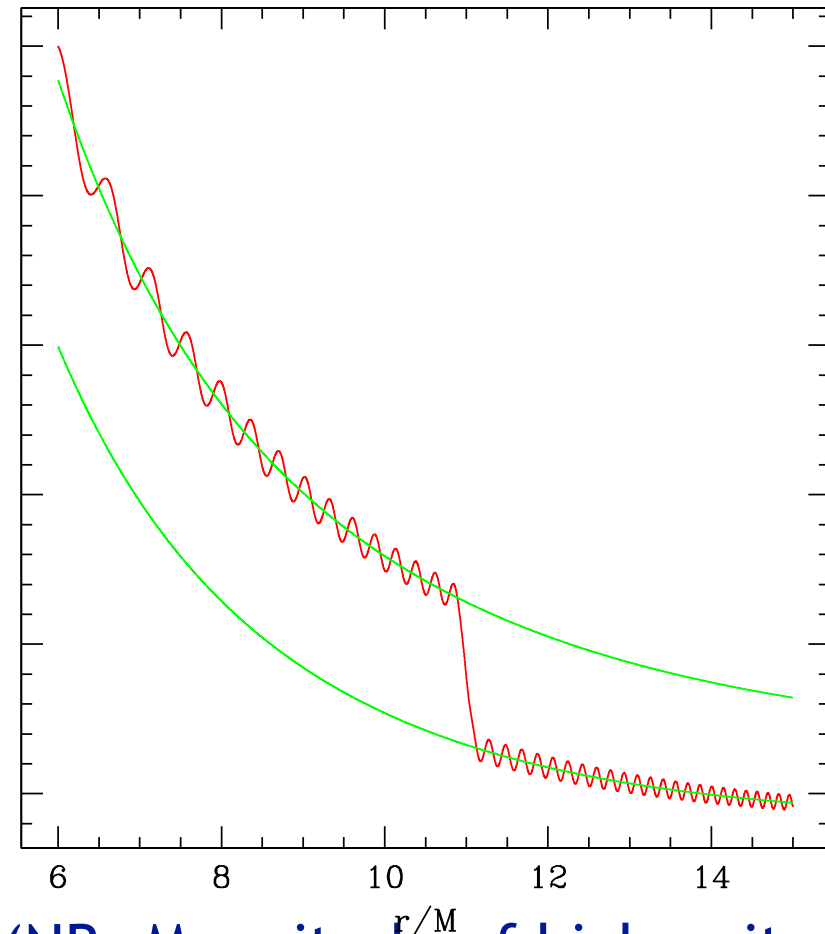


# Examples of issues

Resonances: Each resonance passage “kicks” EMRI.  
Simpler model works well before and after kick.

Evolution in Kerr including resonances may be quite close to a model which does not include resonances but has structure beyond Kerr/GR.

Critical to understand correlations among these effects, biases introduced by including additional physics & parameters.



(NB: Magnitude of kick quite exaggerated in this figure.)



# Examples of issues

Small body spin: Smaller body is not a featureless point mass! Its spin (and other properties) couple to spacetime; it precesses, feels additional “forces.”

$$\frac{DS^{\alpha\beta}}{d\tau} = 0 \qquad F_S^\alpha = -\frac{1}{2}R^\alpha{}_{\nu\lambda\sigma}u^\nu S^{\lambda\sigma}$$

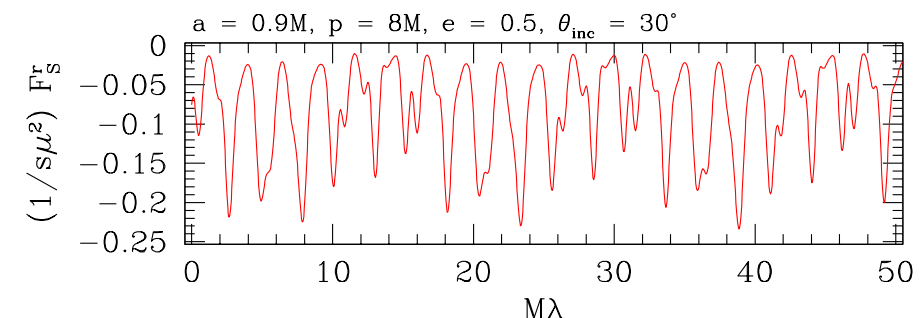
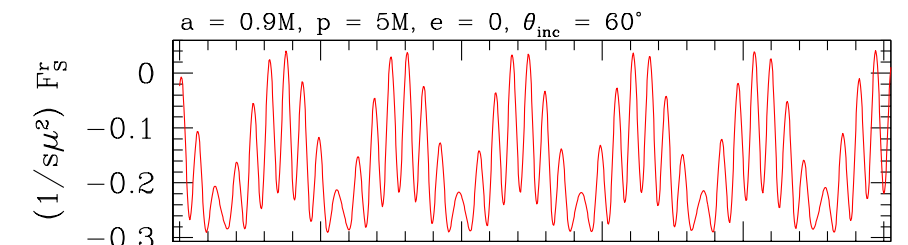
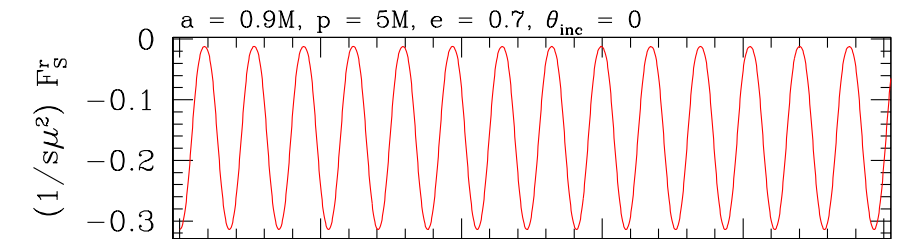
(Linearizing in the spin tensor, which is of order  $\mu^2$  if the small object is itself a black hole.)

# Examples of issues

Small body spin: Smaller body is not a featureless point mass! Its spin (and other properties) couple to spacetime; it precesses, feels additional “forces.”

Precession modulates amplitude at level of mass ratio ... probably below what “matters.”

Spin-curvature coupling force is comparable to other effects we want to measure!



[From Ruangsri, Vigeland, and Hughes,  
PRD **94**, 044008 (2016).]

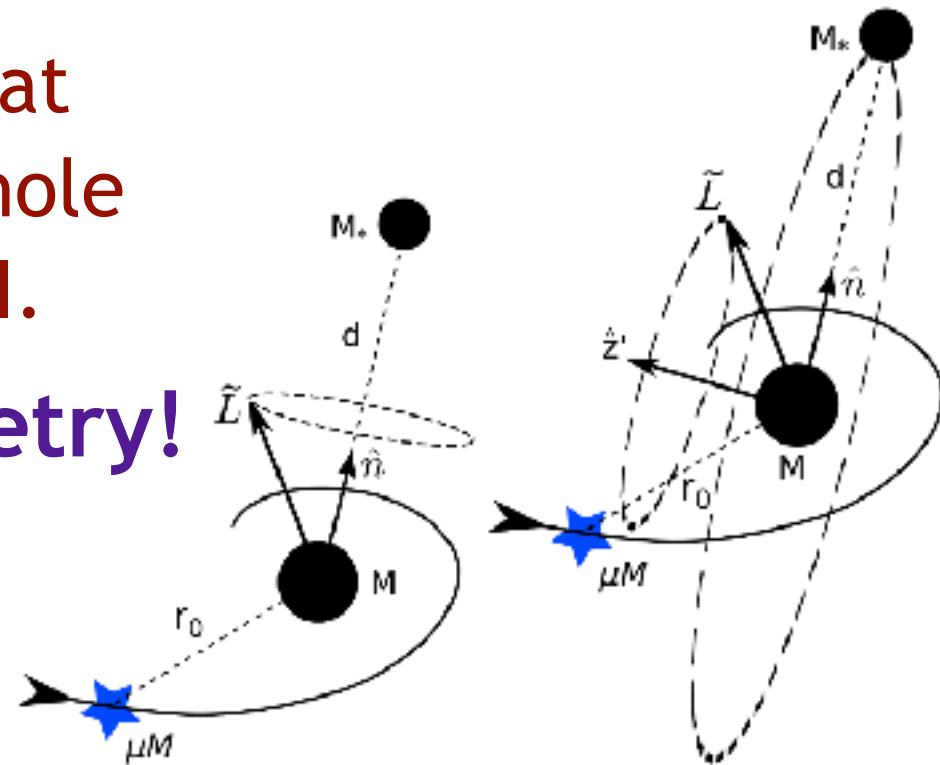
# Examples of issues

Other bodies: A third body near an EMRI will distort its spacetime ... exerting an influence that can change the properties of orbits and inspirals.

Yang & Casals examined what happens if a  $10^7$  Msun black hole is 0.1 parsecs from an EMRI.

3rd body breaks axisymmetry!

Perturbation of 3rd is particularly strong due to resonances between  $r$  and  $\varphi$  motions ... causes precession of EMRI's orbital plane.

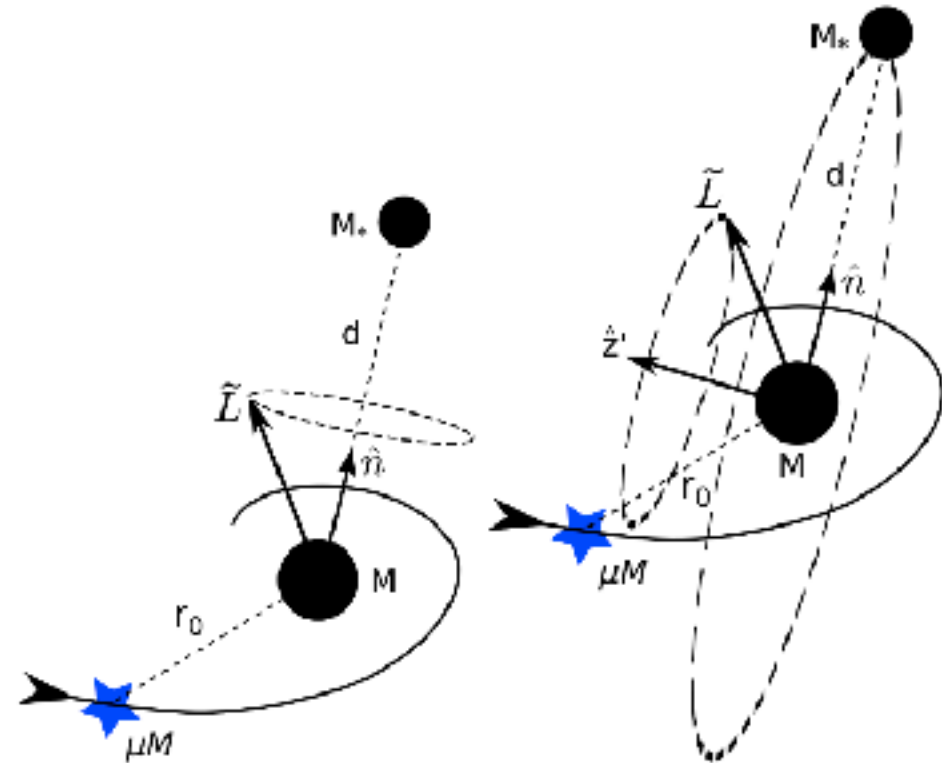


[From Yang and Casals, PRD **96**, 083015 (2017).]

# Examples of issues

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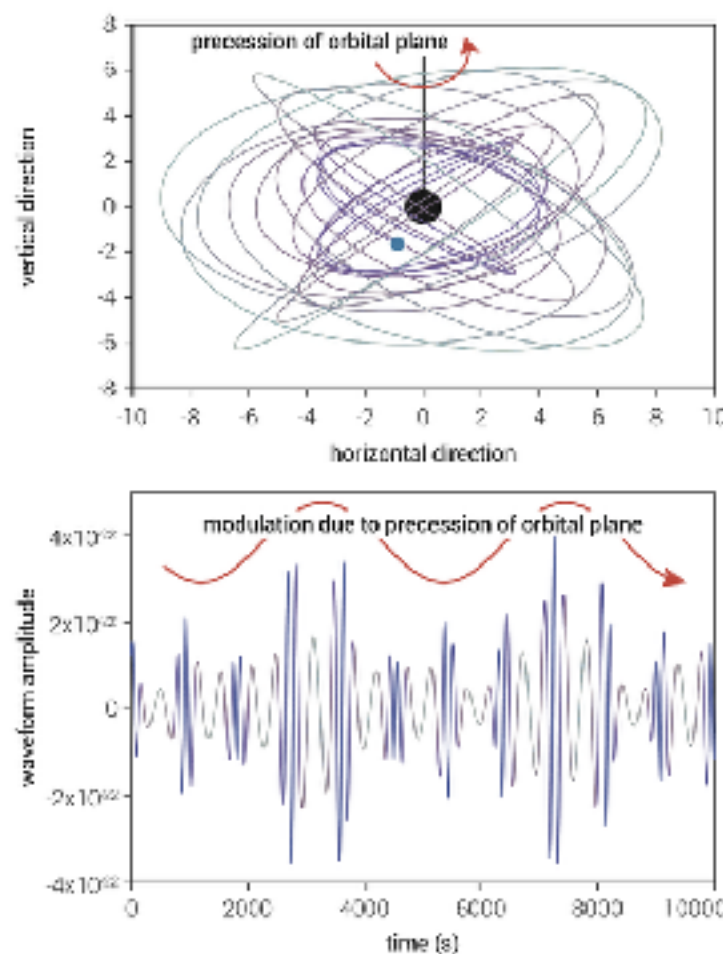
Tide from a  $10^7$  Msun body at 0.1 parsecs is identical to tide from a 10 Msun body at 0.001 parsecs ...  
0.001 parsecs = 1 light day.



**Our galactic center has several stars of this mass which come within a light day of Sgr A\*.**

Clear road to demonstrating that LISA  
can achieve the EMRI science that  
has long been claimed ...

**BUT:** a lot of work must be  
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